Stability of shallow water jets in coastal waters

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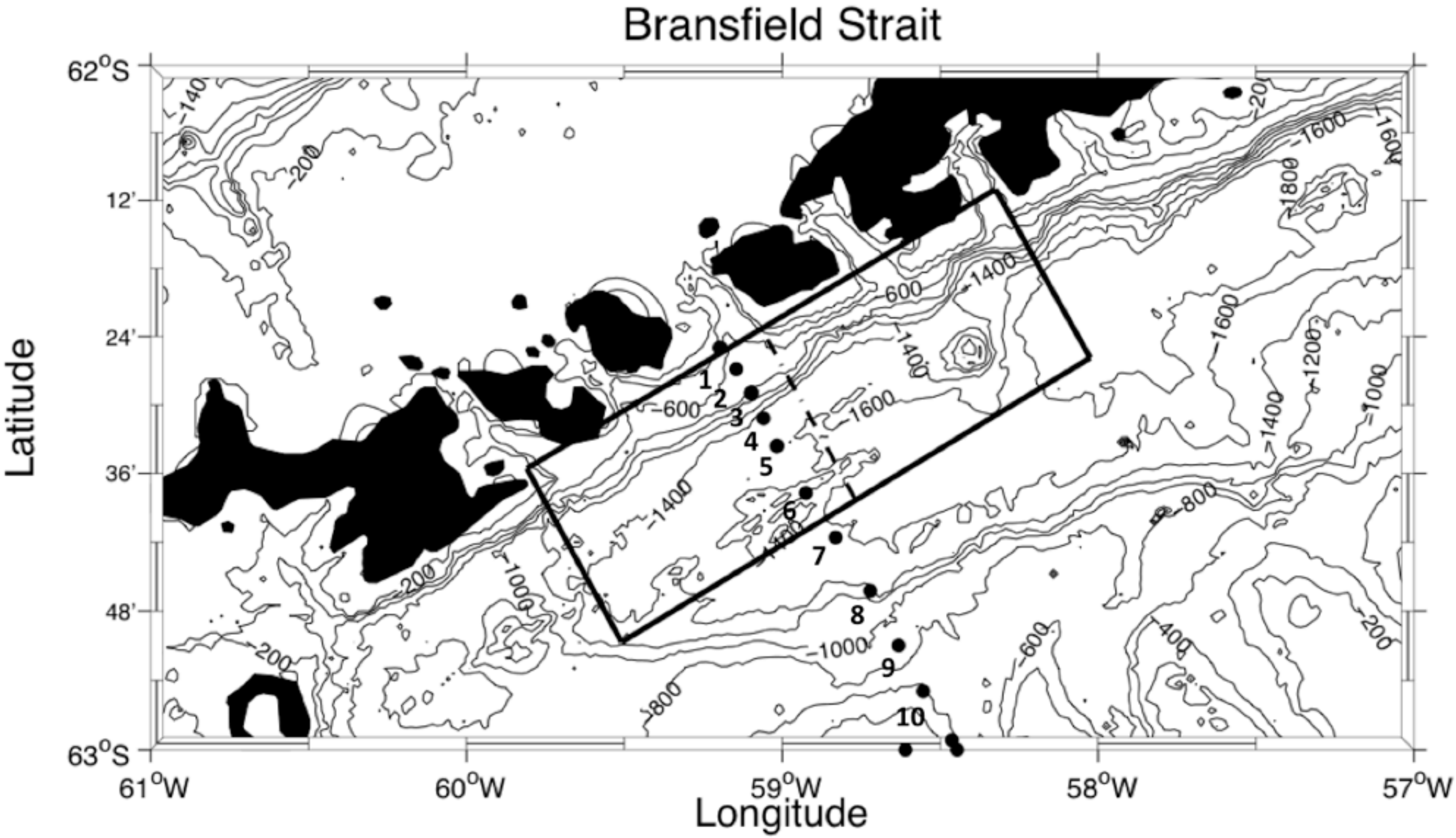
Outline

1. Motivation
2. Model
3. Flat Bottom
4. Topography
5. Conclusions

Mesoscale Jets

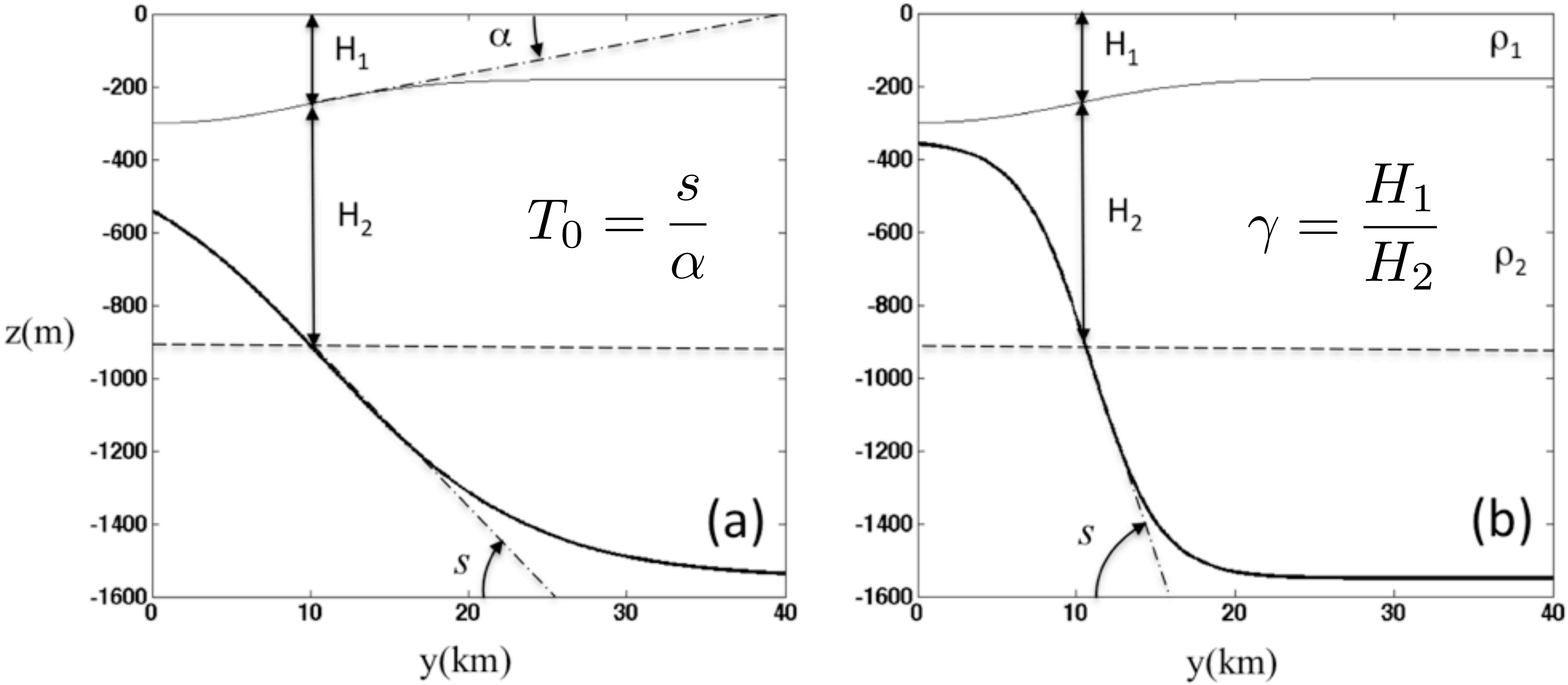
- Winds force oceanic gyres
- Synoptic gyres have mesoscale WBCs
- WBCs generate many vortices
- Mesoscale the most energetic length scale
- Transfers between synoptic and submesoscale
- QG is often used for mesoscale dynamics
- SW has gravity waves and unbalanced motions
- We use SW model to study mesoscale jets
- 2-layers captures some baroclinic features

Bransfield Current



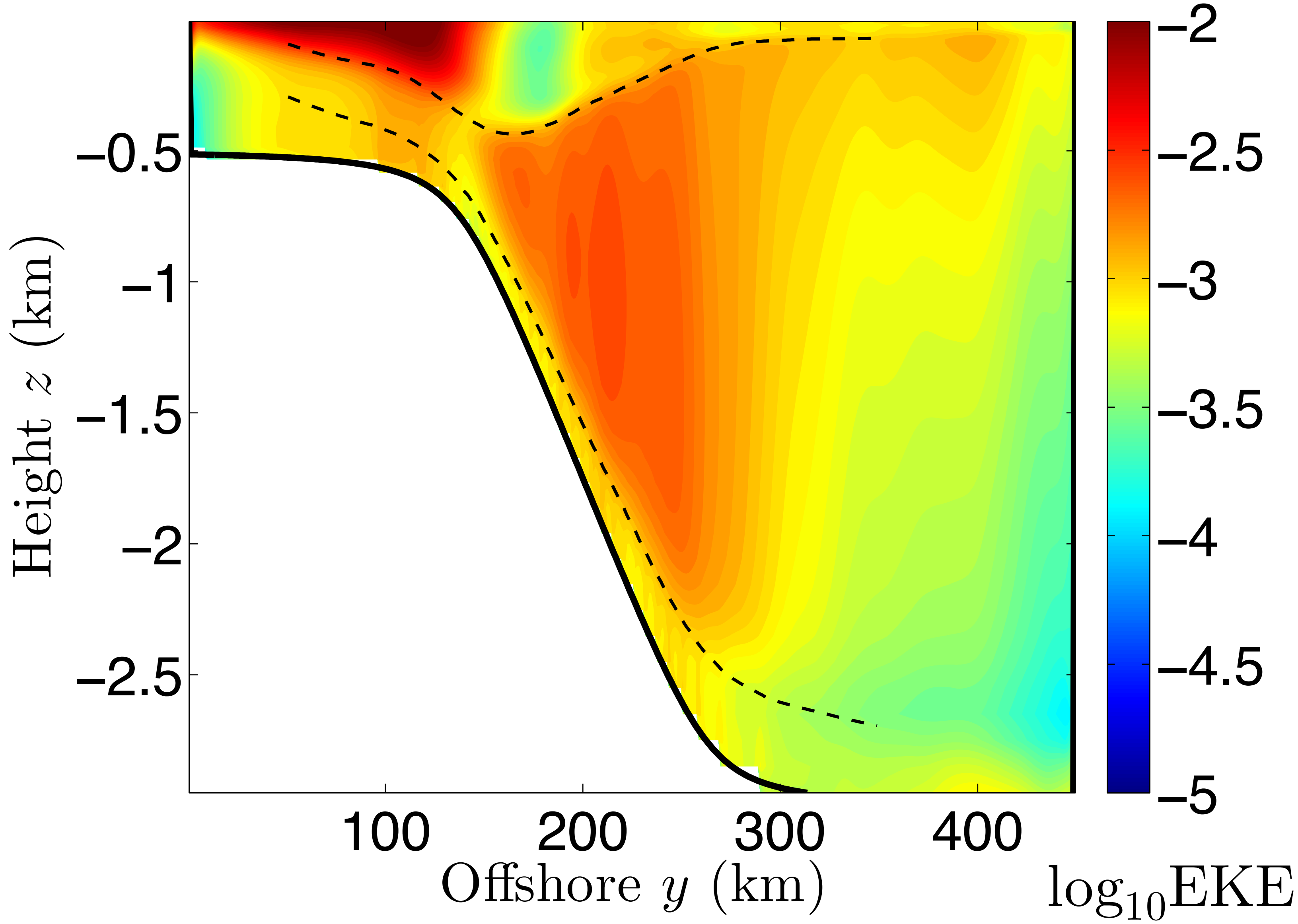
- Very stable due to topography

Bransfield Current



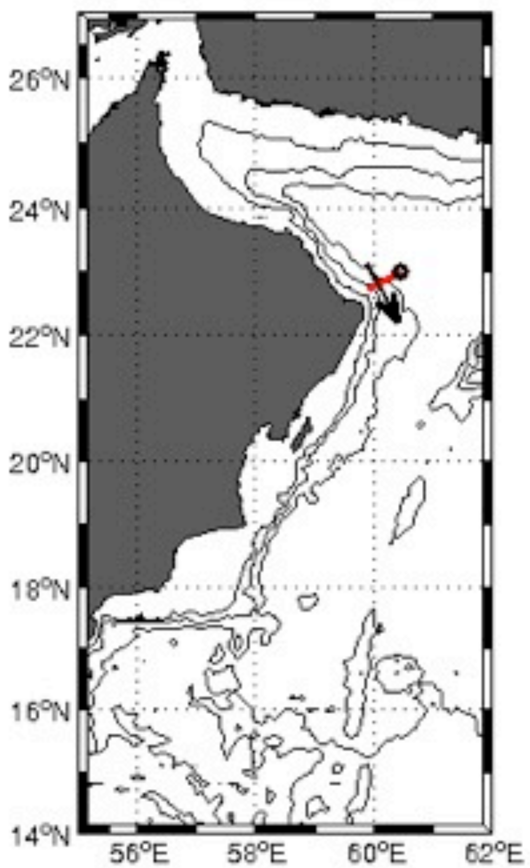
- We approximated it as a 2-L system
- Topography is stabilizing (Poulin et al. 2014)
- Now investigating 3D structure

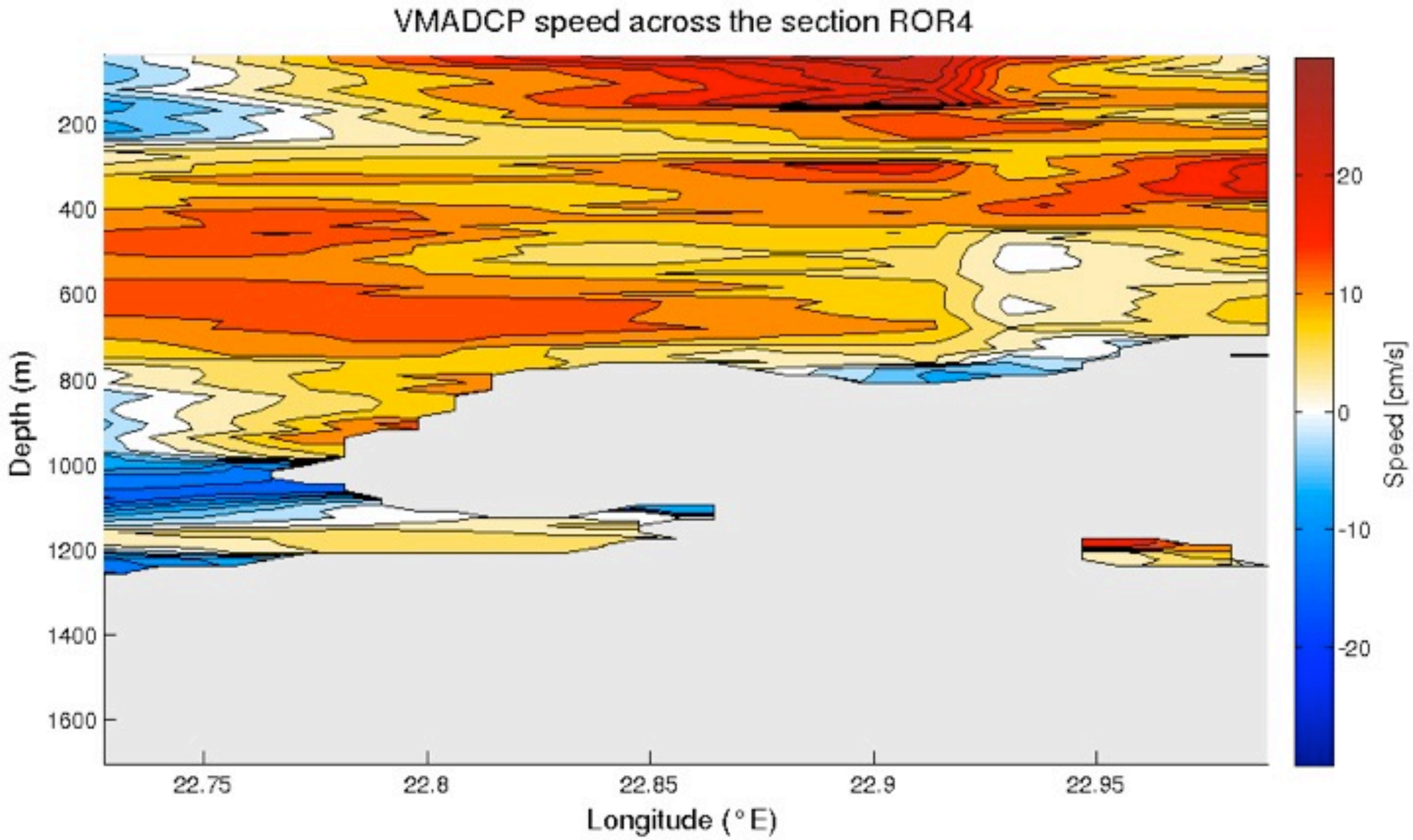
Antarctic Slope Front Water



- Stewart and Thompson have studied ASF
- Investigating a 3-layer approximation
- What can we learn from linear stability?
- Can we explain the cross shelf transport?

Gulf of Oman





- Strong surface and bottom current
- Using a 3-layer SW model
- Current can change a lot along the shelf

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Shallow Water Model

- Governing SW equations with i=1,2

$$\partial_t u_i + \vec{u}_i \cdot \nabla u_i - f v_i = -\partial_x \left(g\eta_1 + \delta_{i2} g' \eta_2 \right),$$
$$\partial_t v_i + \vec{u}_i \cdot \nabla v_i + f u_i = -\partial_y \left(g\eta_1 + \delta_{i2} g' \eta_2 \right),$$
$$\partial_t h_i + \nabla \cdot (h_i \vec{u}_i) = 0,$$

- Describes the motion of fluid columns
- Assume f-plane dynamics
- Includes both Barotropic and Baroclinic motions

Basic State

- Define BT and BC fields

$$\vec{u}_{BT} = \frac{H_1\vec{u}_1 + H_2\vec{u}_2}{H_1 + H_2}, \quad \vec{u}_{BC} = \vec{u}_1 - \vec{u}_2.$$

- Motivates defining the following

$$Ro_{BT} = \frac{U_{BT}}{fL}, \quad Ro_{BC} = \frac{U_{BC}}{fL}, \quad Bu_{BT} = \frac{gH}{f^2L^2}, \quad Bu_{BC} = \frac{g'H}{f^2L^2}.$$

- Basic state

$$\begin{aligned}\overline{u}_i =& U_i \operatorname{sech}^2\left(\frac{y}{L_j}\right), \\ \overline{\eta}_1 =& -H\left(\frac{Ro_1}{Bu_{BT}}\right)\tanh\left(\frac{y}{L_j}\right), \\ \overline{\eta}_2 =& H\left(\frac{Ro_1}{Bu_{BC}} - \frac{Ro_2}{Bu_{BT}}\right)\tanh\left(\frac{y}{L_j}\right).\end{aligned}$$

Simple QG Model

- Consider a 5 patch QG model n=1,2,3,4,5

$$\left(\frac{\partial}{\partial t} + U_n \frac{\partial}{\partial x}\right)\left[\nabla^2 \psi_n - (-1)^n L_d^{-2}\left(\psi_2 - \psi_1\right)\right] + \frac{dQ_n}{dy}\frac{\partial \psi}{\partial x} = 0,$$

$$\frac{dQ_n}{dy} = -\frac{d^2 U_n}{dy^2} - (-1)^n L_d^{-2}(U_1 - U_2),$$

- Reduces to the BVP

$$(U_n - c)\left\{\frac{d^2\hat{\psi}_n}{dy^2} - k^2\hat{\psi}_n - (-1)^n L_d^{-2}(\hat{\psi}_2 - \hat{\psi}_1)\right\} + \frac{dQ_n}{dy}\hat{\psi}_n = 0.$$

$$\left[\frac{\hat{\psi}_n}{U_n - c}\right]_{y_0} = 0, \quad \text{and} \quad \left[(U_n - c)\frac{d\hat{\psi}_n}{dy} - \hat{\psi}_n \frac{dU_n}{dy}\right]_{y_0} = 0.$$

- Exact eigenvalue problem is 16x16. Fast to solve!

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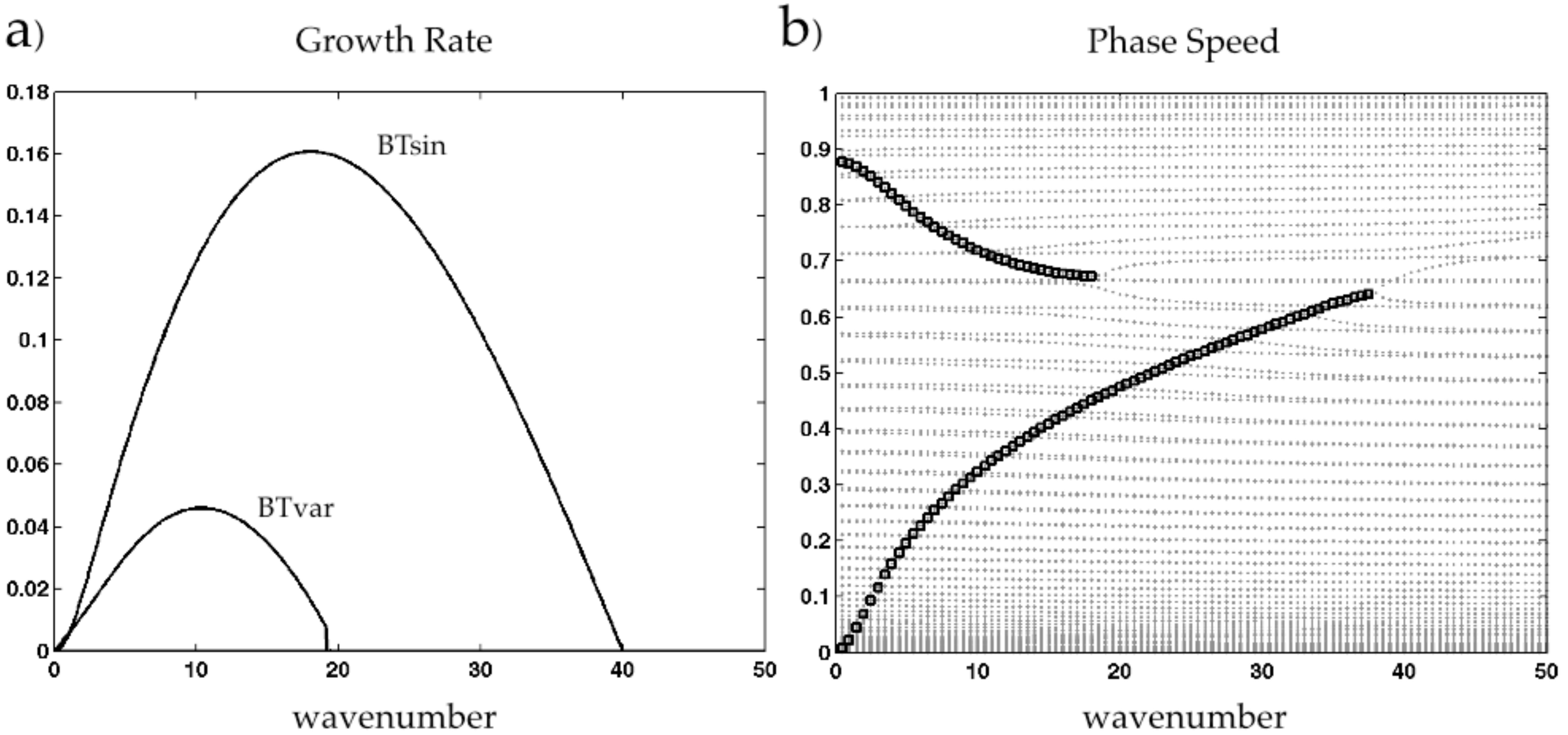
One-Layer Growth Rates

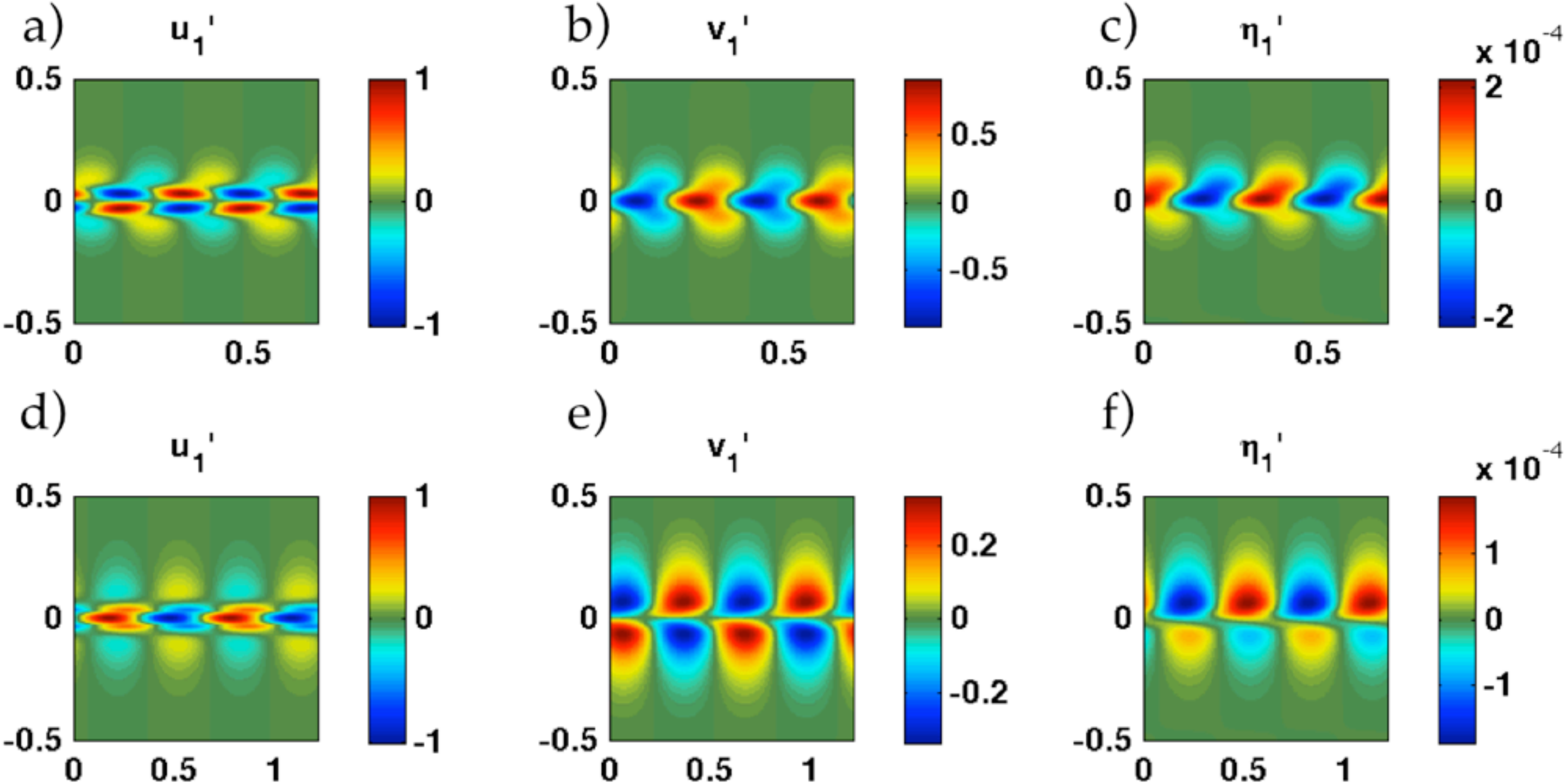
- Linear stability of the 1-layer case
- Poulin and Flierl (2003)



- Sinuous and Varicose modes
- 2-Layer results in Irwin and Poulin (2014)

One-Layer Modal Structures

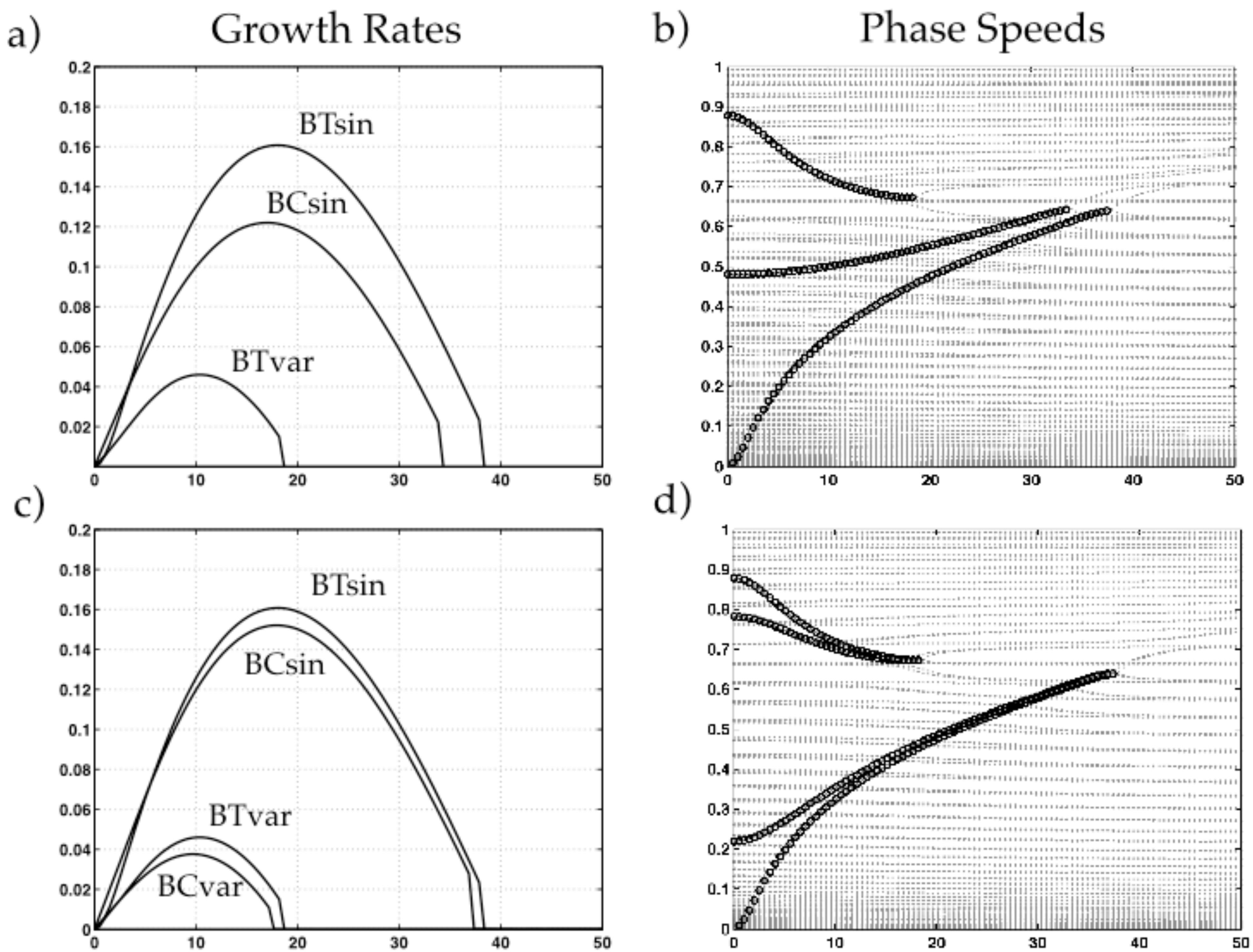
- a) sinuous and b) varicose



- Modes agree with classical theory
- No strong asymmetries

Two-Layer Barotropic Flow

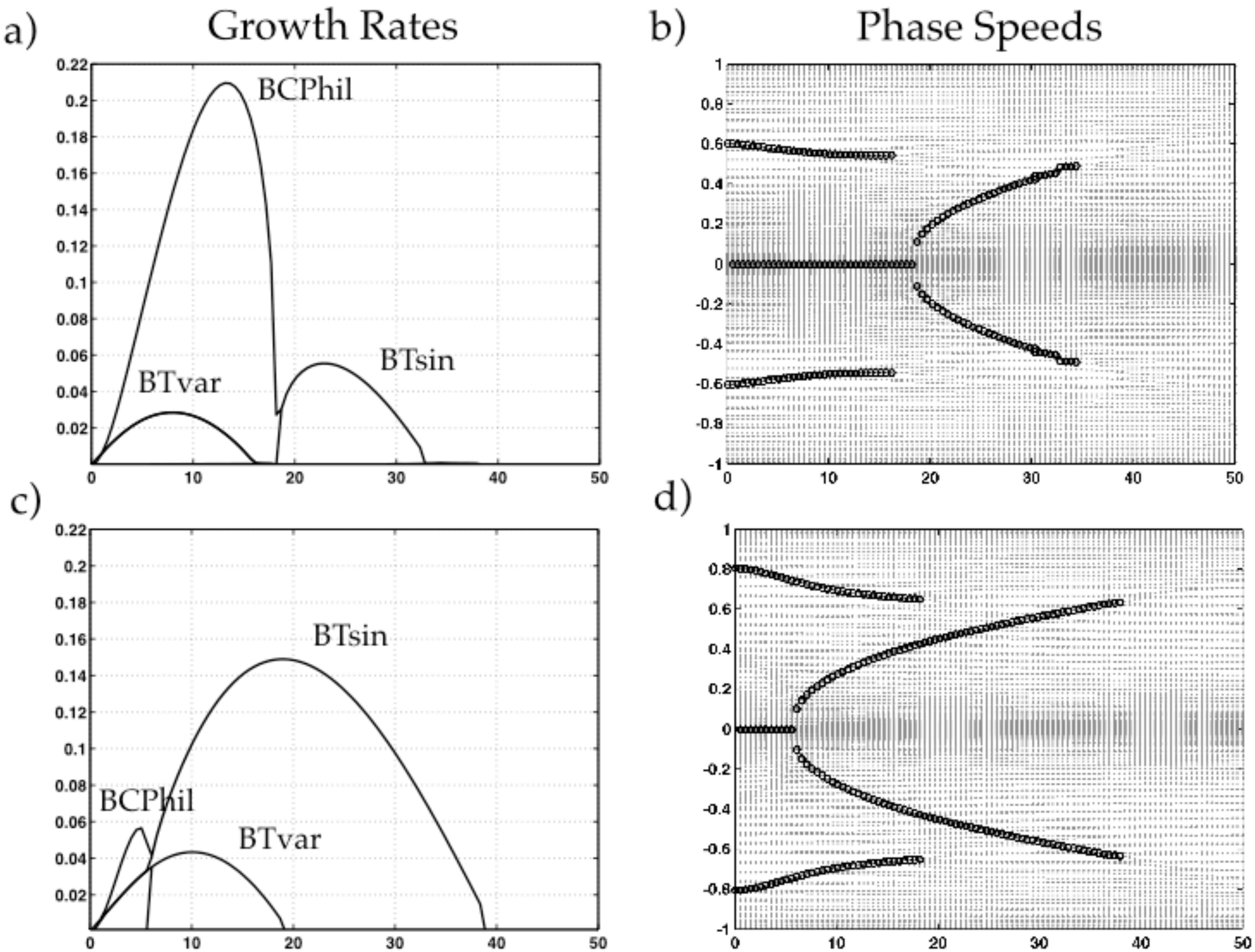
- Uniform flow in both layers: g′=0.1 and g′=1



- Weaker stratification stables the system

Two-Layer Baroclinic Flow

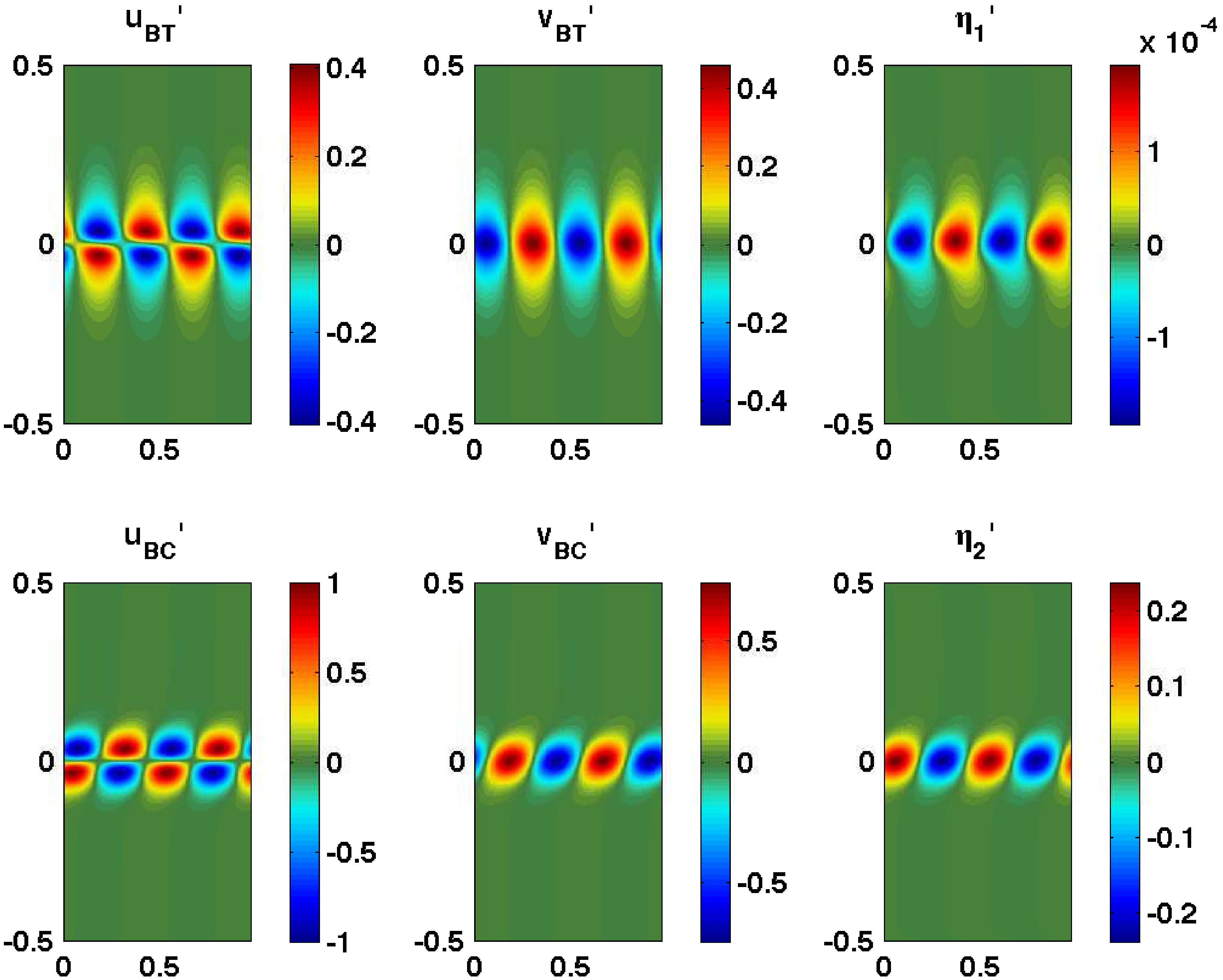
- Opposite flow in the two layers



- Have 5 potentially unstable modes

Two-Layer Baroclinic Flow

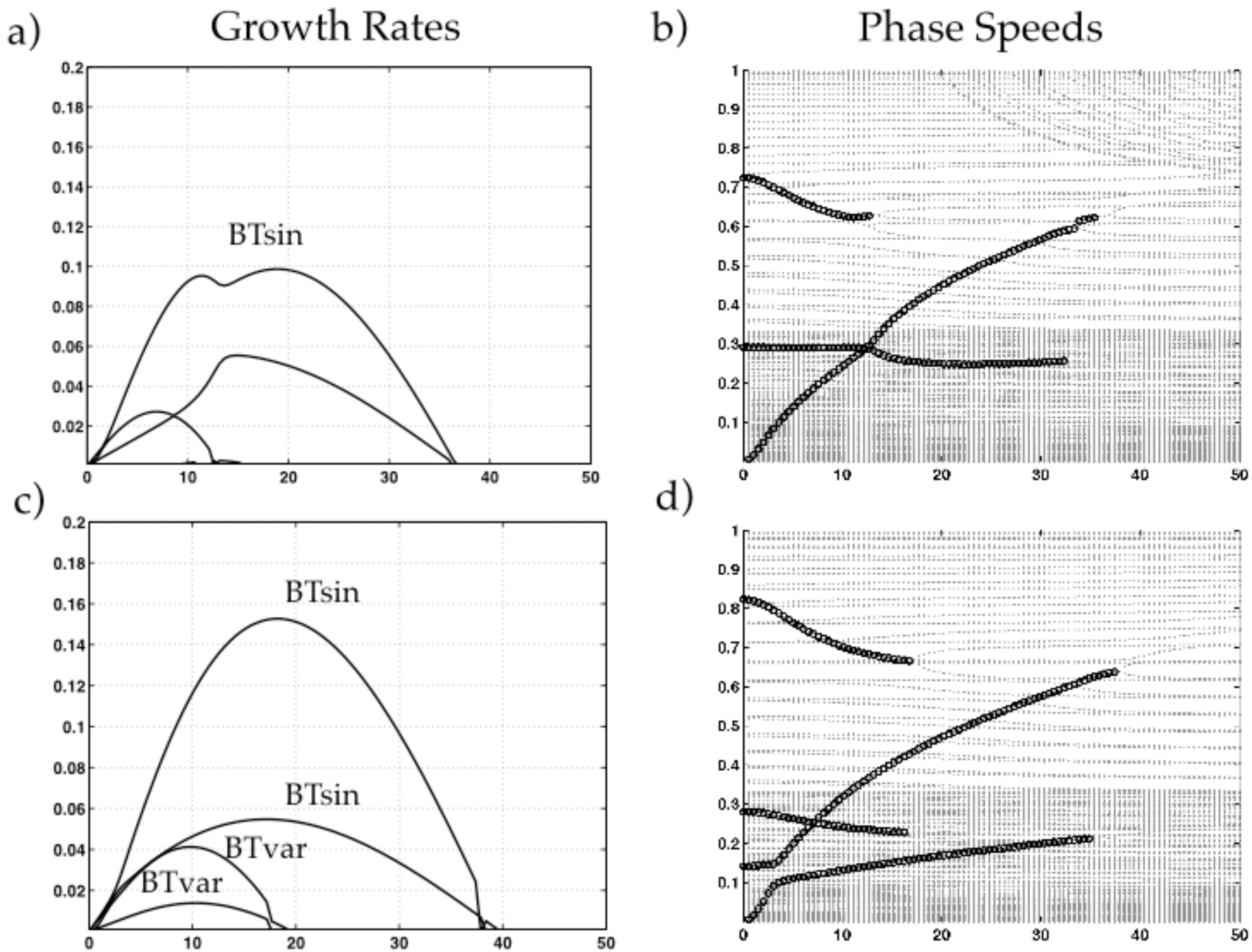
- Classical Phillips Mode



- Stronger BC field

Two-Layer Mixed BT-BC Flow

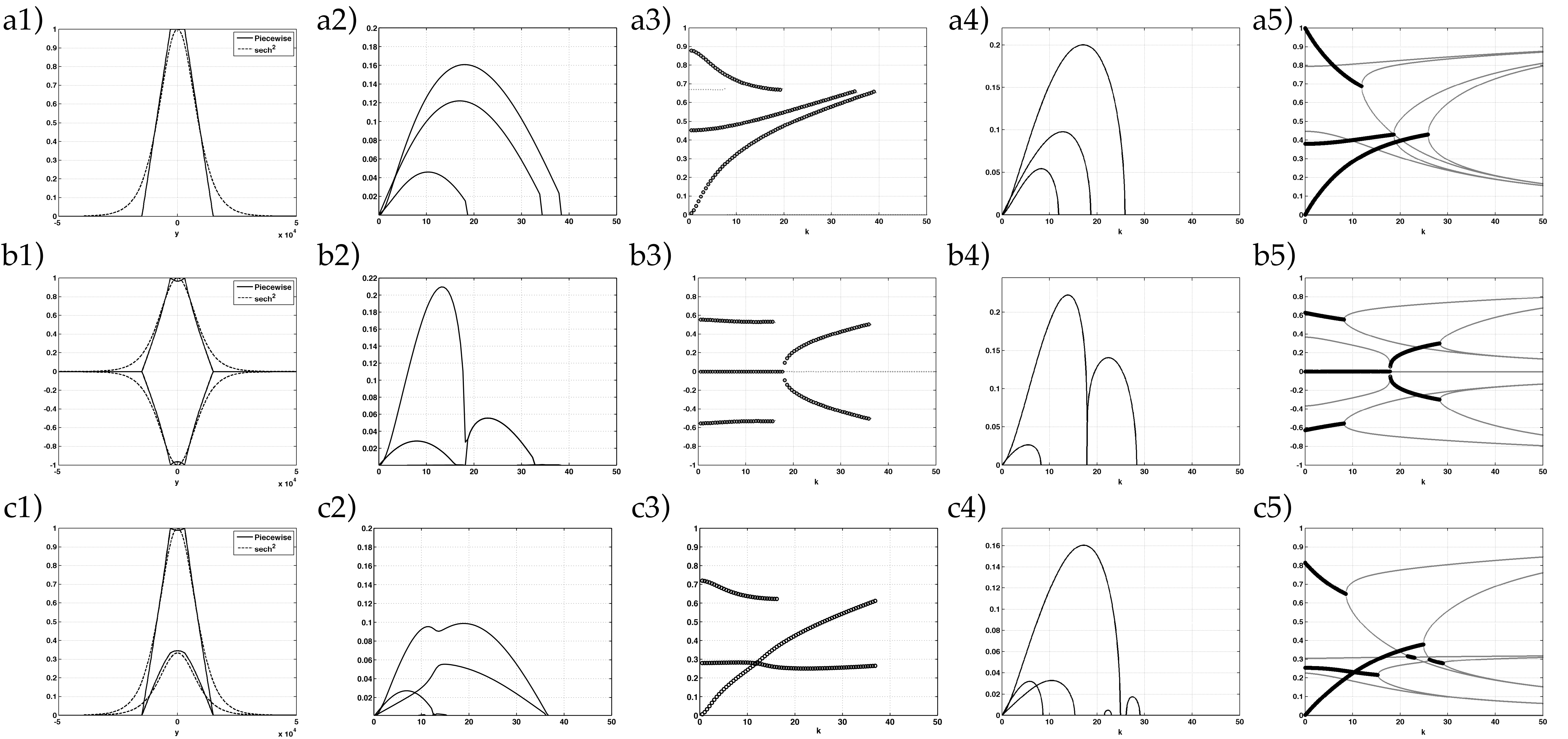
- Mixture of previous two cases



- Complicated and hard to classify

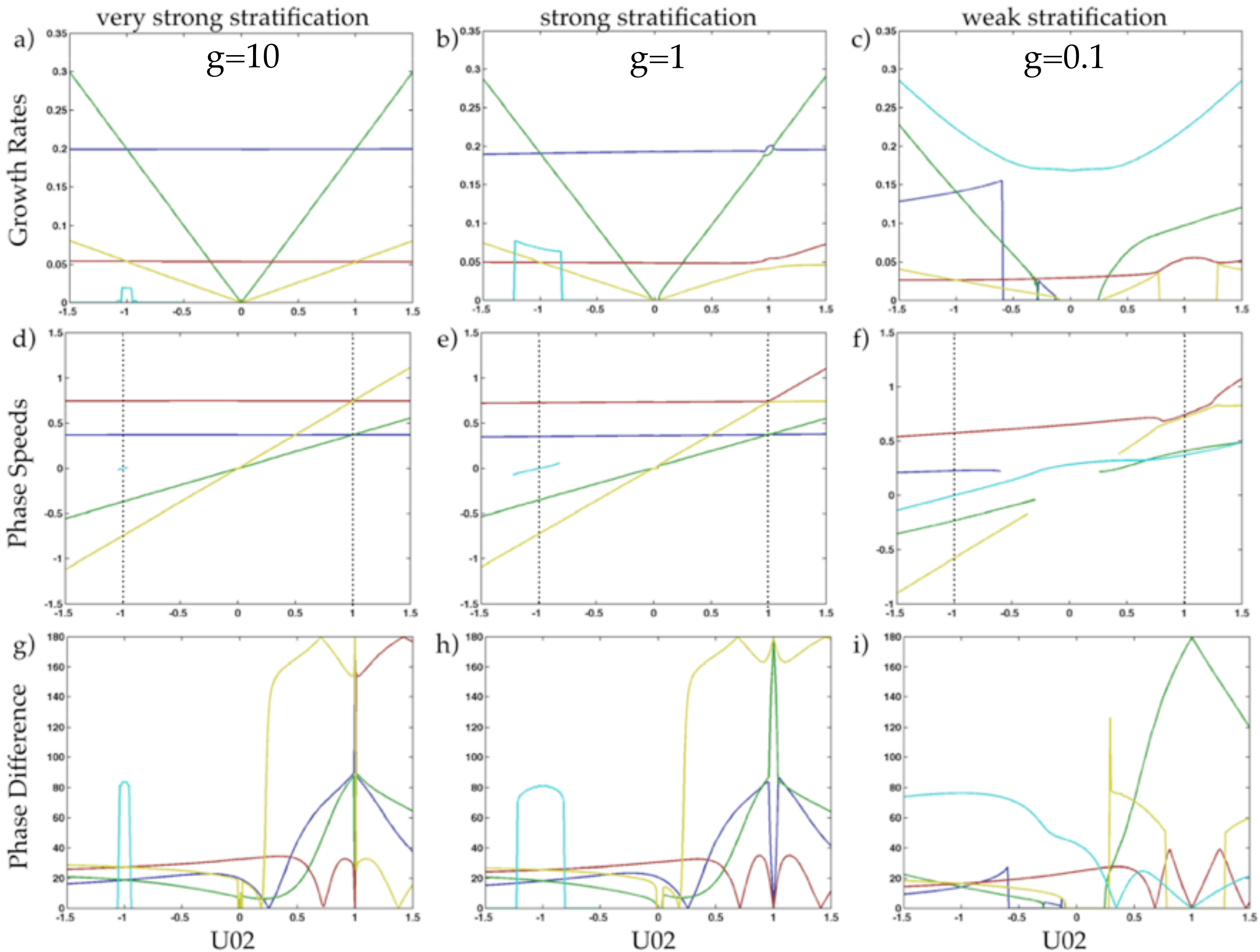
5 Patch QG Model

- Profiles, growth rates and phase speeds

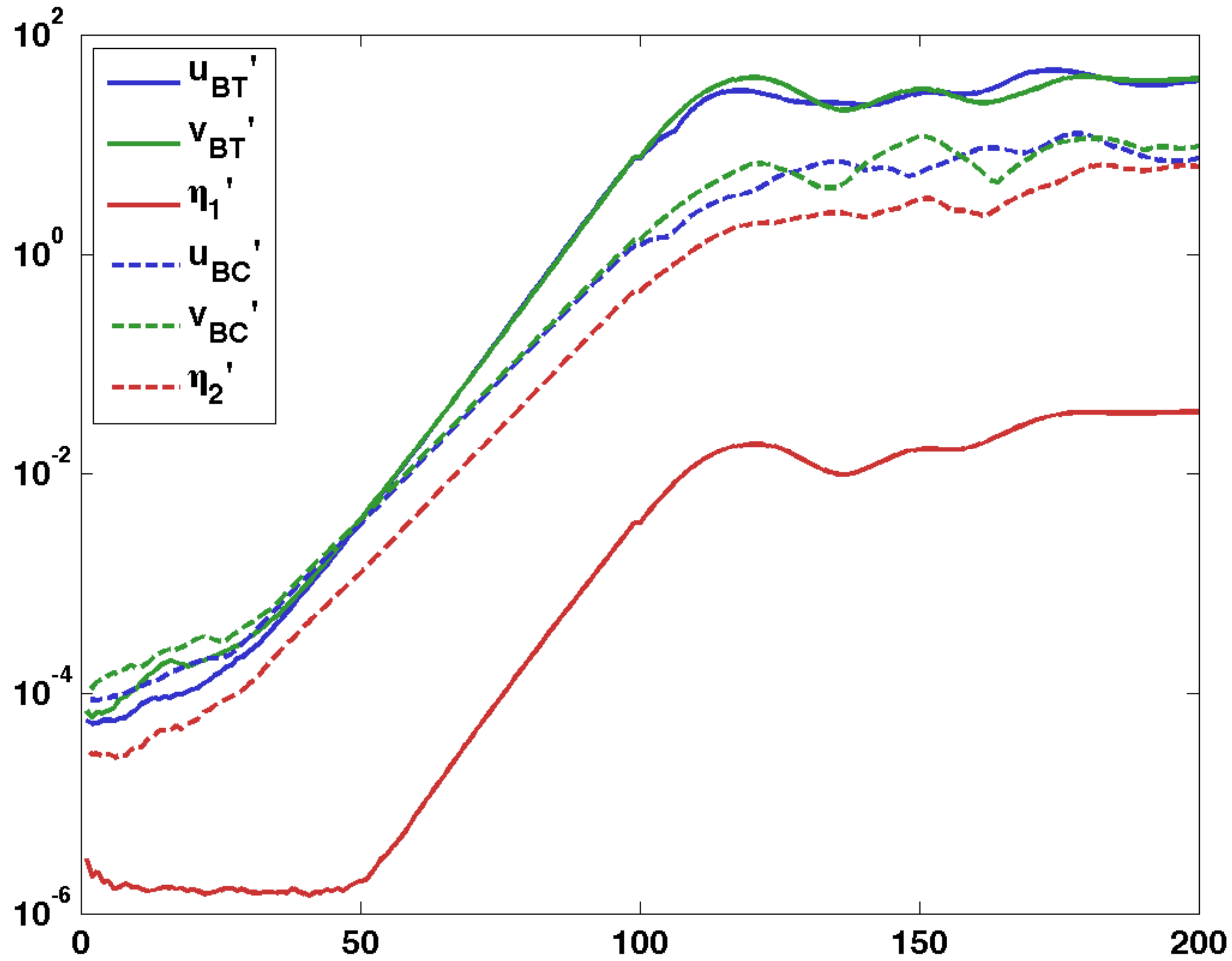


- Profiles look similar and modes are similar too
- Growth rates similar but lengths are off

5 Patch QG Model



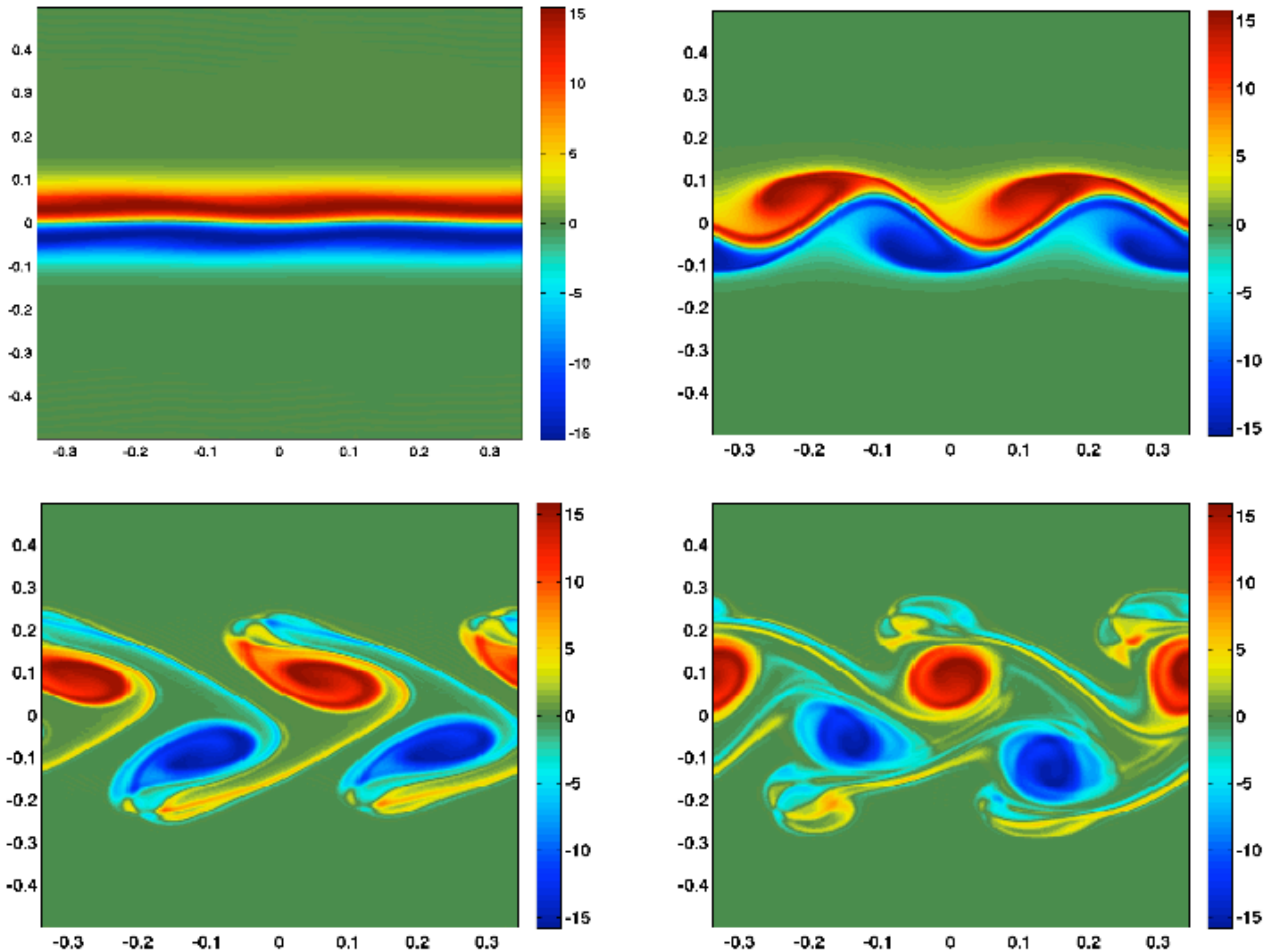
Nonlinear Evolution of 2L BT



- Initially both BT and BC modes grow
- Look at BT and BC fields to compute growth
- BT mode grows faster
- Get nonlinear interactions at equilibration

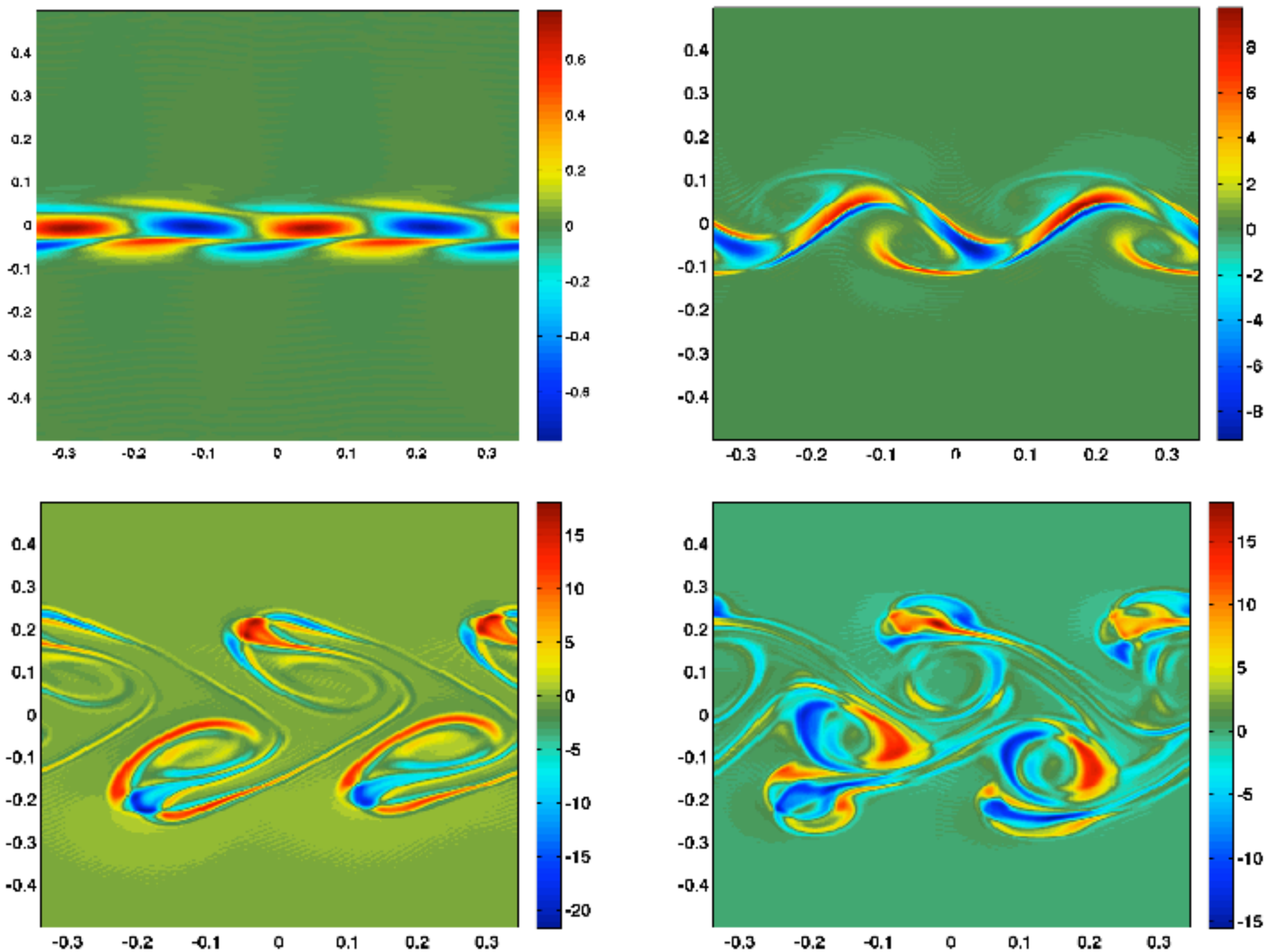
Nonlinear Evolution of 2L BT

- Barotropic vorticity



Nonlinear Evolution of 2L BT

- Baroclinic vorticity



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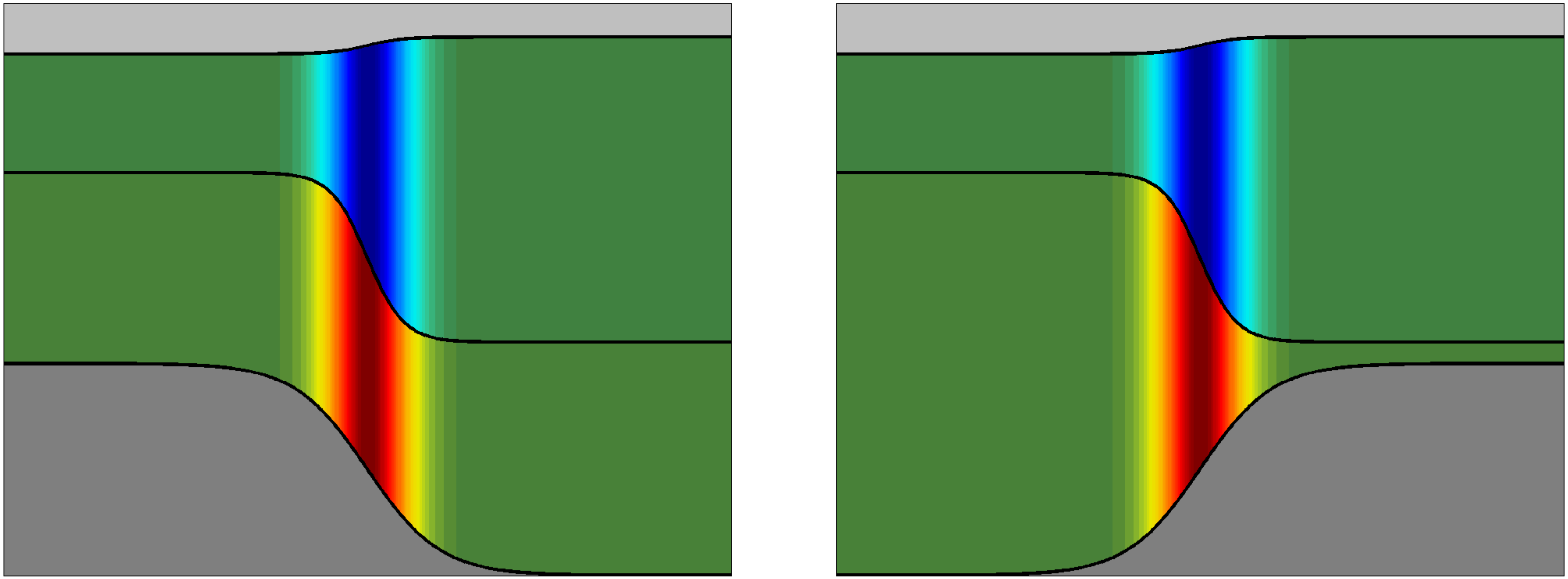
Two-Layer Flow over Topography

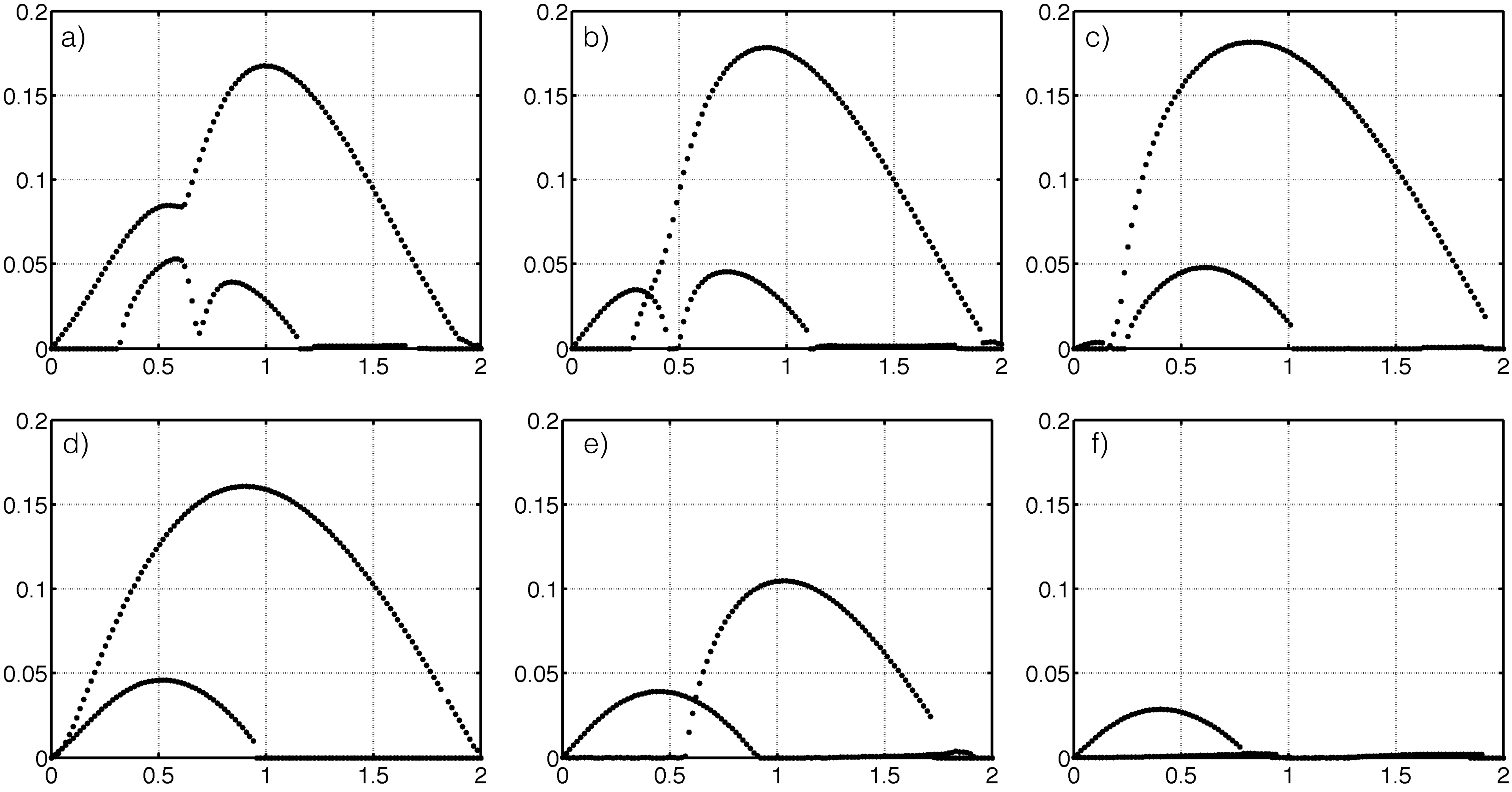
- Prograde and Retrograde Flows



- 1-layer case in Poulin and Flierl (2005)
- We finally got around to the 2-layer case

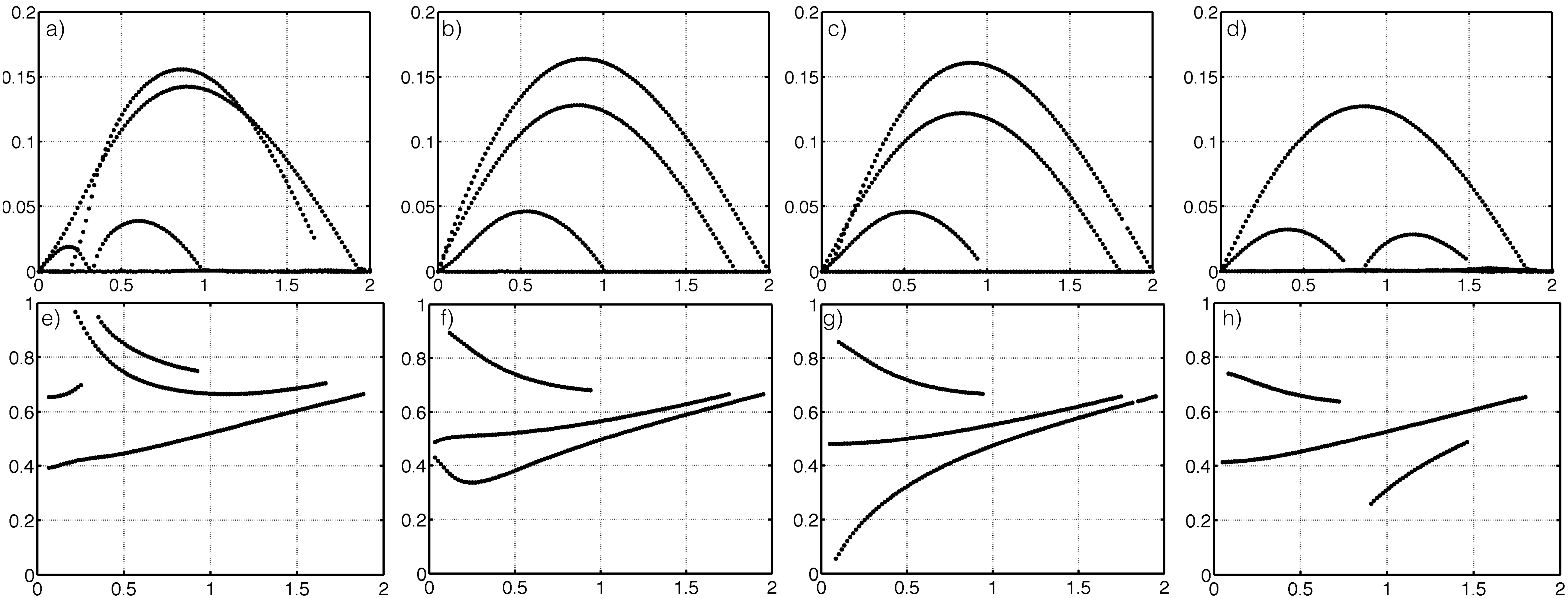
One-Layer Flow

- To = -475, -400, -225, 0, 225, 475
- New topographic mode for prograde flow



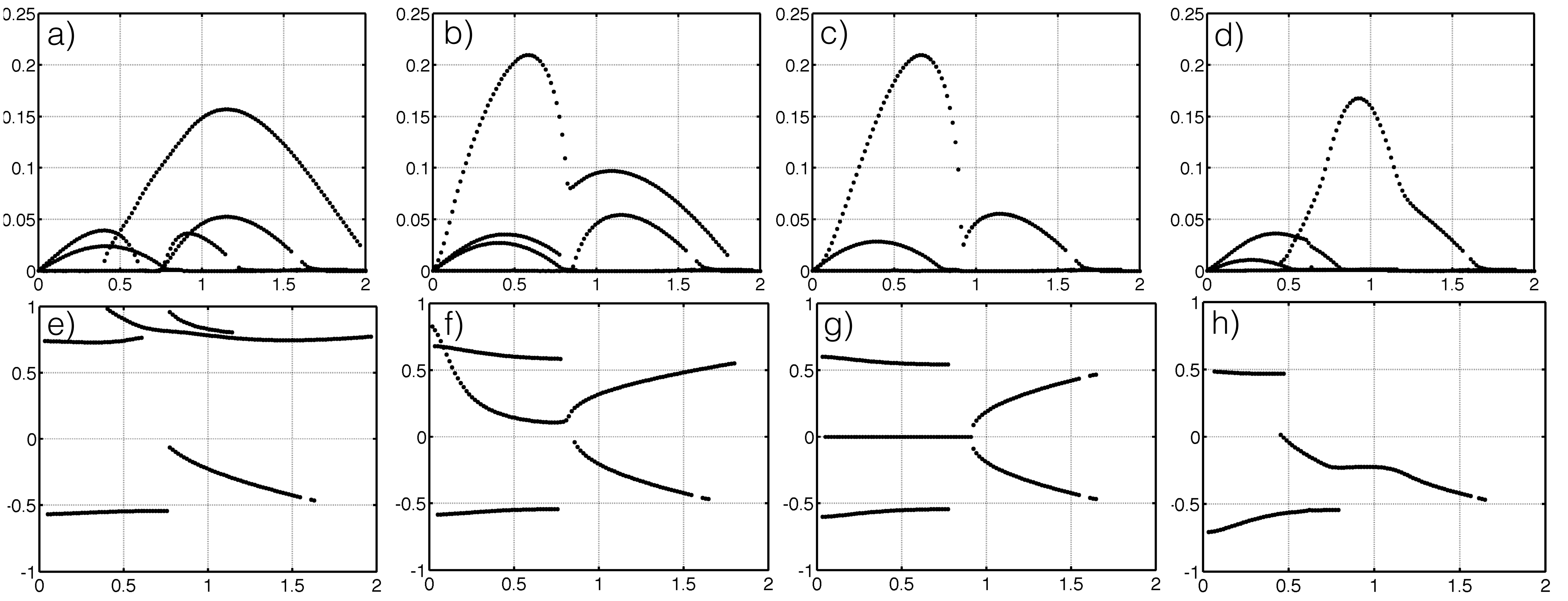
Two-Layer BT Flow

- To = -225, -50, 0, 225
- Retrograde stabilizing
- Prograde can be destabilizing or stabilizing



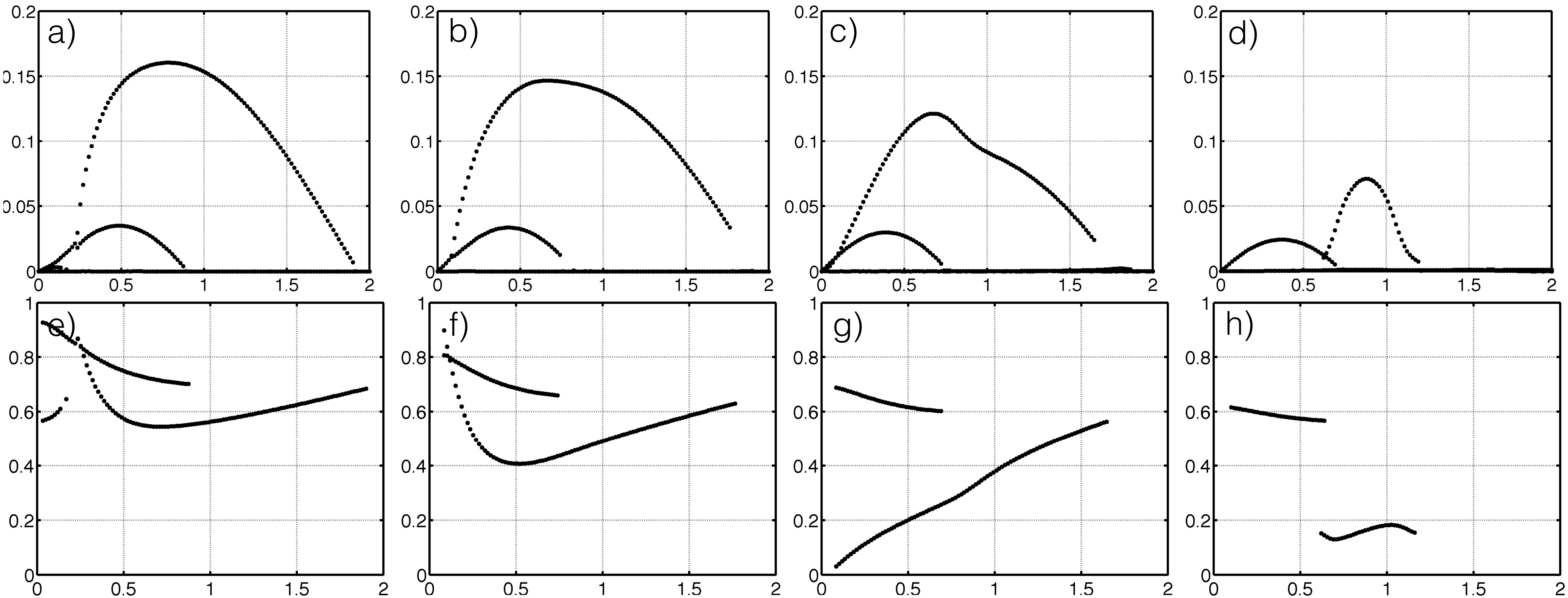
Two-Layer BC Flow

- To = -325, -50, 0, 125
- Retrograde stabilizing
- Prograde can be destabilizing or stabilizing



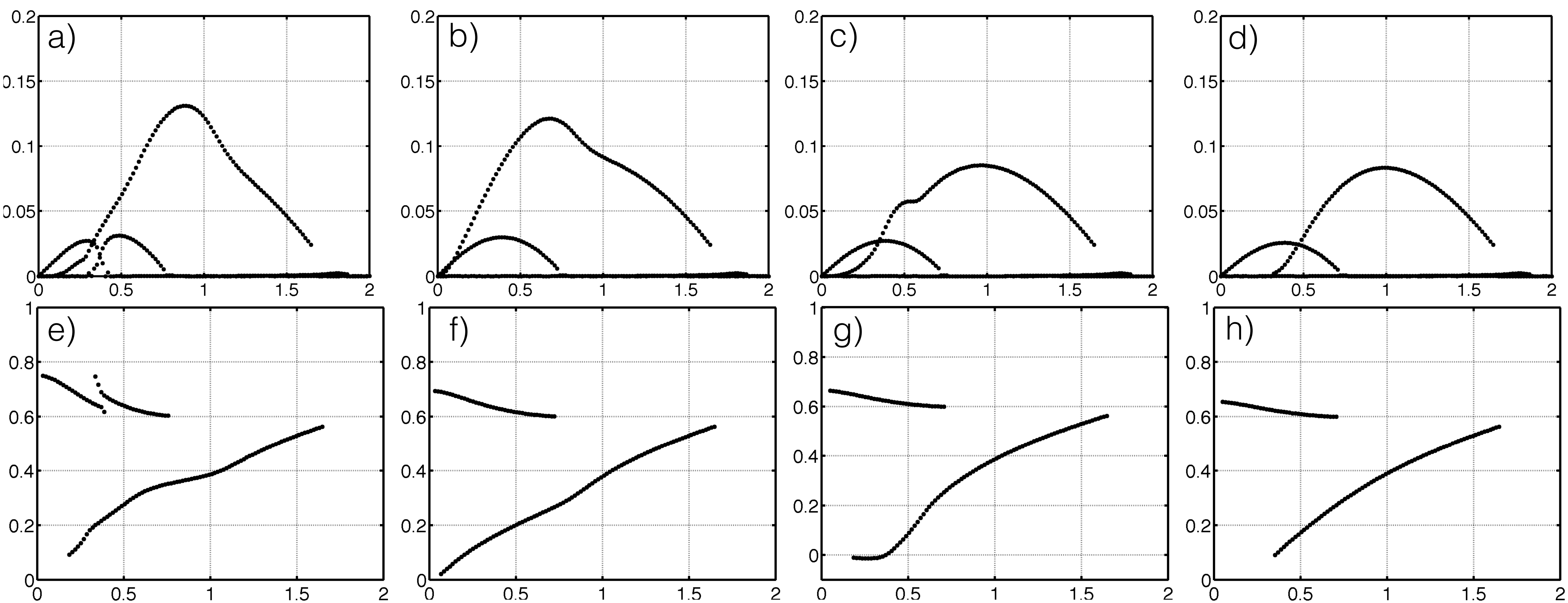
Two-Layer Bottom intensified Flow

- To = -225, -125, 0, 125
- Retrograde stabilizing
- Prograde can be destabilizing or stabilizing



Two-Layer Surface intensified Flow

- To = -125, 0, 125, 225
- Retrograde stabilizing
- Prograde can be destabilizing or stabilizing



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Conclusions

- Can apply 2- or 3-L SW models to many problems
- Flat bottom
  - 5 unstable modes can occur
  - Switching from bottom/top to BT/BT
  - Strong dependency on stratification
  - Hard to capture the not most unstable modes
- Topography
  - Retrograde stabilizing (all modes)
  - Prograde is both destabilizing and stabilizing
  - Transition value depends on Burger number
  - A topographic mode develops
- Must look at nonlinear problem in more detail
- Will compare with 3D simulations