Visioconference (from Toulouse)
for TOSCA Irriga-Detection
22 Mai 2017

De la modélisation du chaos à la classification des couverts agricoles

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CESBIO
UMR 5126
(UPS-CNES-CNRS-IRD)

lefe
Inphyniti
PNTS

Theory of nonlinear dynamical systems

- Henri Poincaré

probably the first one to understand

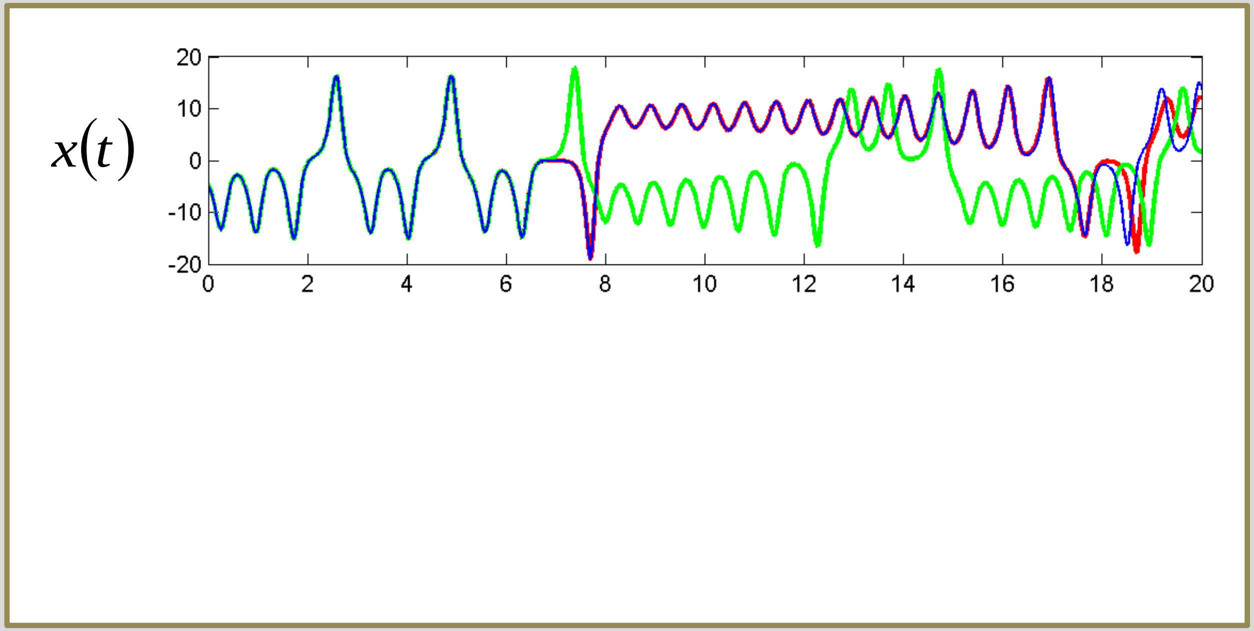
how a system can be both

Deterministic & unpredictable at long term

Lorenz system (1963)

$$\begin{cases} \dot{x} = \sigma\left(y - x\right) \\ \dot{y} = -xz + \rho x - y \\ \dot{z} = xy - \beta z \end{cases}$$

$x(t)$



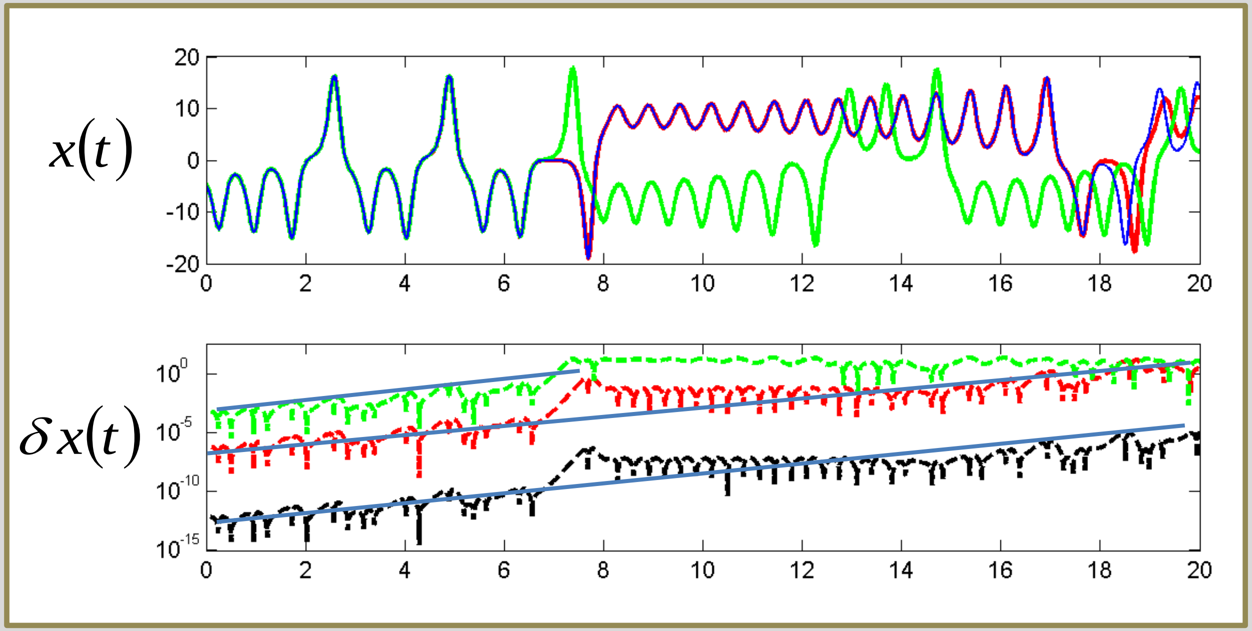
Same deterministic equations
&
Small difference i.c.

Very different time evolution

Lorenz system (1963)

$$\begin{cases} \dot{x} = \sigma\left(y - x\right) \\ \dot{y} = -xz + \rho x - y \\ \dot{z} = xy - \beta z \end{cases}$$

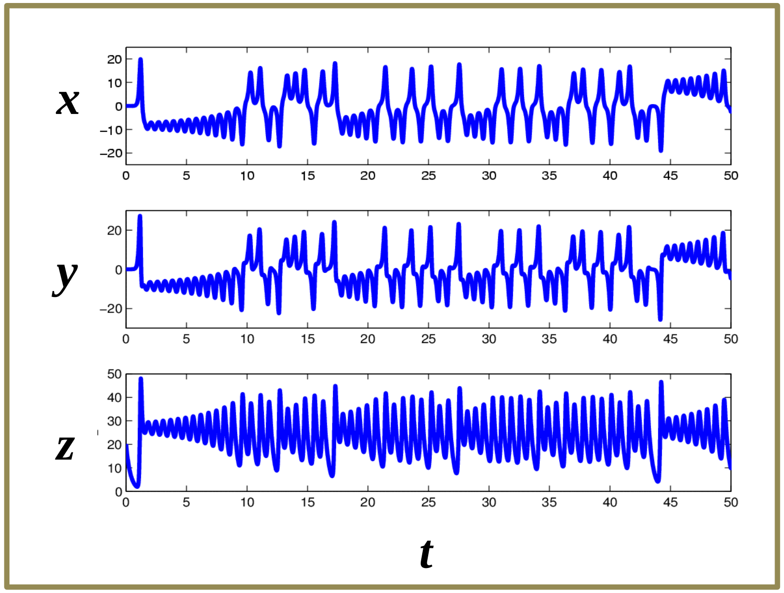
$x(t)$



Lorenz system (1963)

$$
\begin{cases}
\dot{x} = \sigma (y - x) \\
\dot{y} = -xz + \rho x - y \\
\dot{z} = xy - \beta z
\end{cases}
$$

$$x(t),\ y(t),\ z(t)$$



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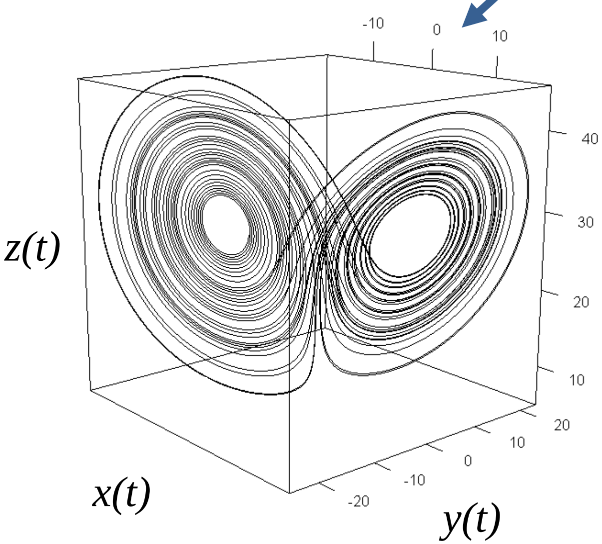
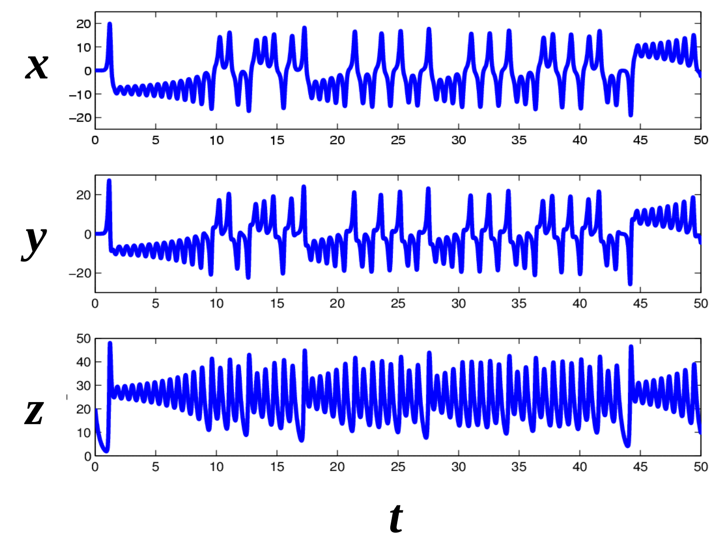
\end{cases}
$$

$$x(t),\ y(t),\ z(t)$$





The *Phase Space* (or State space) :

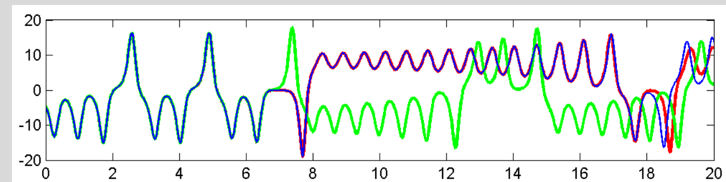
a space that provides the complete solutions

Theoretical background

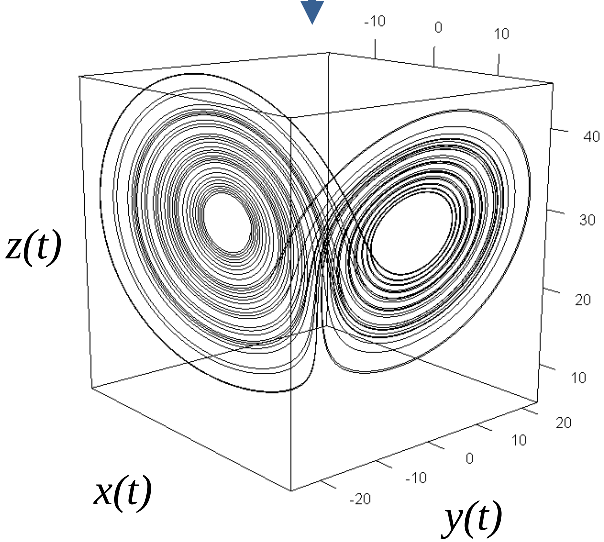
Takens Theorem (1981)

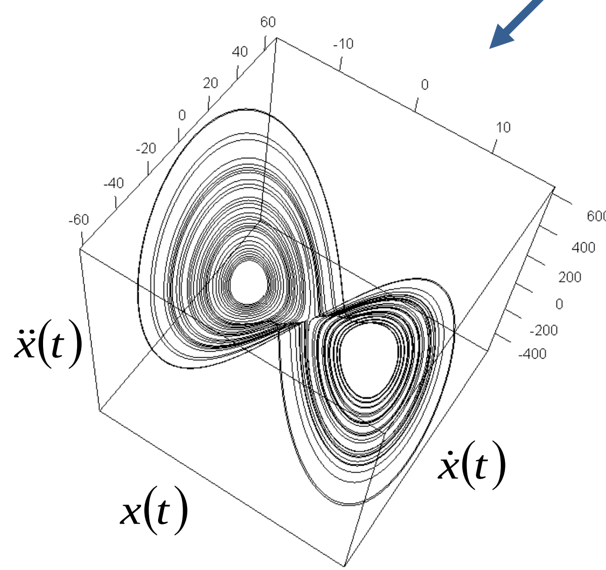
$$\begin{cases} \dot{x} = \sigma(y - x) \\ \dot{y} = -xz + \rho x - y \\ \dot{z} = xy - \beta z \end{cases}$$

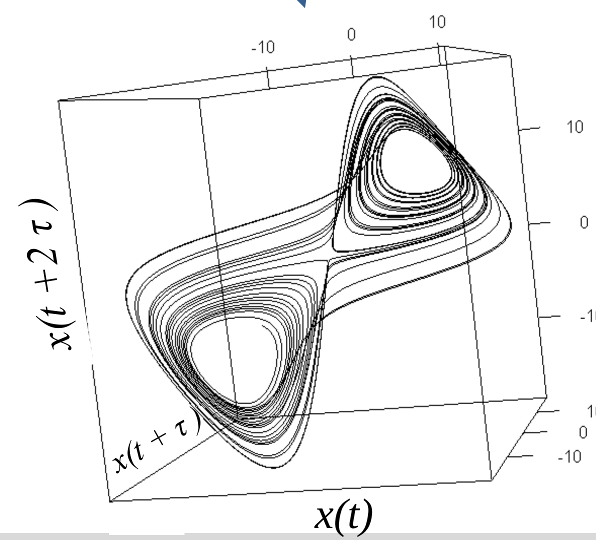
$x(t)$



$x(t),\ y(t),\ z(t)$







F. Takens

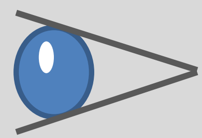
Nonlinear invariants are conserved

Global modeling

- multivariate

$$
\begin{cases}
\dot{x}_1 = f_1(x_1, x_2, ..., x_n) \\
\dot{x}_2 = f_2(x_1, x_2, ..., x_n) \\
\quad \dots \\
\dot{x}_n = f_n(x_1, x_2, ..., x_n)
\end{cases}
$$

x_i is observed



?

Equations from one single observed variable?

J. Crutchfield

Crutchfield & McNamara (1987)

Global modeling

G. Gouesbet

Gouesbet (1991)

- multivariate

$$\begin{cases} \dot{x}_1 = f_1(x_1, x_2, .., x_n) \\ \dot{x}_2 = f_2(x_1, x_2, .., x_n) \\ \dots \\ \dot{x}_n = f_n(x_1, x_2, .., x_n) \end{cases}$$

x_i is observed

- univariate

$$\begin{cases} \dot{x}_i = X_2 \\ \dot{X}_2 = X_3 \\ \dots \\ \dot{X}_n = F(x_i, X_2, .., X_n) \end{cases}$$

Global modeling

C. Letellier

G. Gouesbet

Gouesbet & Letellier (1994)

- multivariate

$$\begin{cases} \dot{x}_1 = f_1(x_1, x_2, .., x_n) \\ \dot{x}_2 = f_2(x_1, x_2, .., x_n) \\ \dots \\ \dot{x}_n = f_n(x_1, x_2, .., x_n) \end{cases}$$

x_i is observed

- univariate

$$\begin{cases} \dot{x}_i = X_2 \\ \dot{X}_2 = X_3 \\ \dots \\ \dot{X}_n = F(x_i, X_2, .., X_n) \end{cases}$$

Polynomial approximation

$$F(x_i, X_2, .., X_n) = P(x_i, X_2, .., X_n)$$

Good results for some variables of some systems / but not all …

Global modeling technique

- Empirical approach

  Few *a priori* knowledge required

  Directly applies to time series

- Well adapted to low-dimensional systems

  Can bring strong argument for determinism

  All the conditions and properties of chaos in a consistent fashion

- Strong and rich theoretical background

  Global solutions

  Theory of nonlinear dynamical systems

  (Poincaré’s work, Takens’ Theorem, topology of chaos, etc.)

H. Poincaré

Global modeling

C. Letellier

L. Aguirre

- multivariate

$$\begin{cases} \dot{x}_1 = f_1(x_1, x_2, .., x_n) \\ \dot{x}_2 = f_2(x_1, x_2, .., x_n) \\ \dots \\ \dot{x}_n = f_n(x_1, x_2, .., x_n) \end{cases}$$

« Problem » of observability

- univariate

$$\begin{cases} \dot{x}_i = X_2 \\ \dot{X}_2 = X_3 \\ \dots \\ \dot{X}_n = F(x_i, X_2, .., X_n) \end{cases}$$

Letellier & Aguirre (2001)

Global modeling

C. Letellier

L. Aguirre

- multivariate

$$\begin{cases} \dot{x}_1 = f_1(x_1, x_2, .., x_n) \\ \dot{x}_2 = f_2(x_1, x_2, .., x_n) \\ \ldots \\ \dot{x}_n = f_n(x_1, x_2, .., x_n) \end{cases}$$

« Problem » of observability

- univariate

$$\begin{cases} \dot{x}_i = X_2 \\ \dot{X}_2 = X_3 \\ \ldots \\ \dot{X}_n = F(x_i, X_2, .., X_n) \end{cases}$$

Letellier & Aguirre (2001)

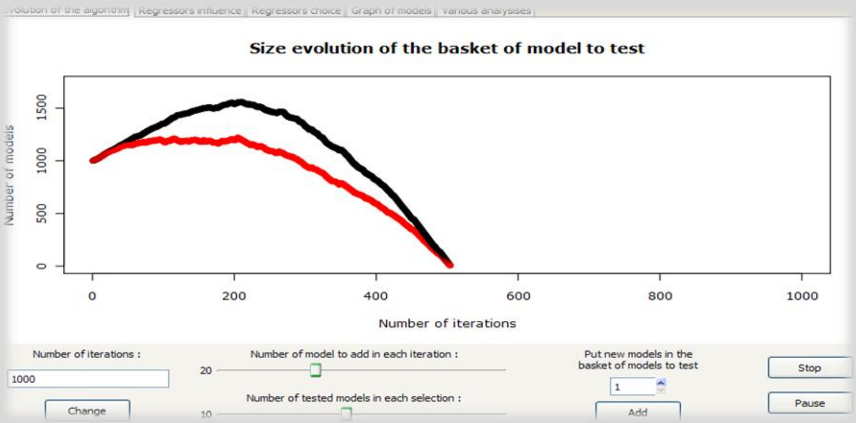
Sophus Lie

Lie derivatives

$$\mathcal{L}_f f_i(\boldsymbol{x}) = \frac{\partial f_i(\boldsymbol{x})}{\partial \boldsymbol{x}} \boldsymbol{f}(\boldsymbol{x}) = \sum_{k=1}^{m} \frac{\partial f_i(\boldsymbol{x})}{\partial \boldsymbol{x}} f_k$$

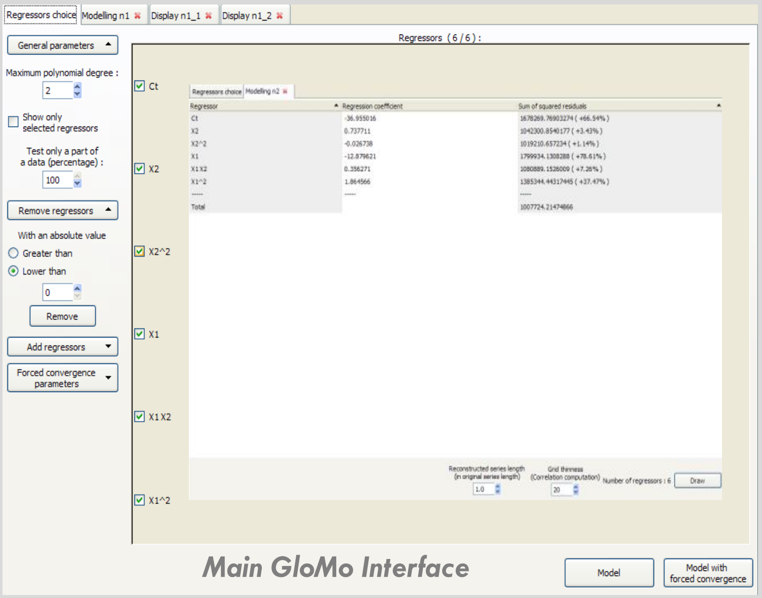
Algorithmic tools *R-packages*

PoMoS



Models selection algorithm interaction

GloMo



Main GloMo Interface

L. Drapeau
Ing. IRD

M. Huc
Ing. CNRS

R. Coudret

F. Le Jean

M. Chassan

Mangiarotti et al.
Phys. Rev. E (2012)

PoMoS & GloMo, the potential of the approach

J. Hudson's group
Fei et al. (1994)

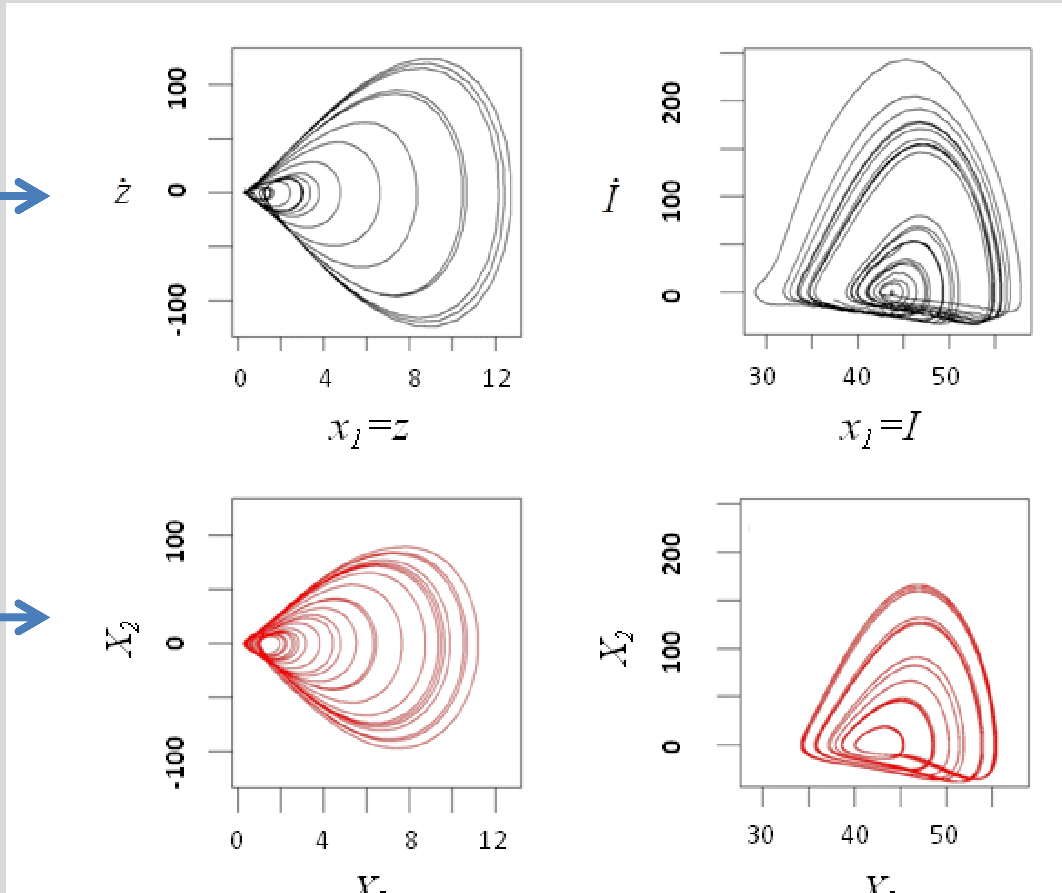
Letellier & Aguirre (2002) ←

Rössler-z
(low observability)

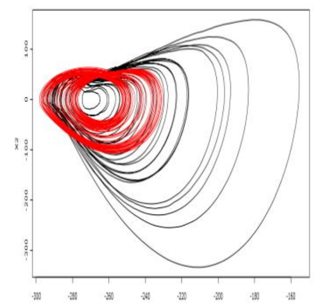
Electrodissolution of Cu
in phosphoric acid

data →

models →



Belousov-Zhabotinski
reaction
(data: F. Argoul 1987)

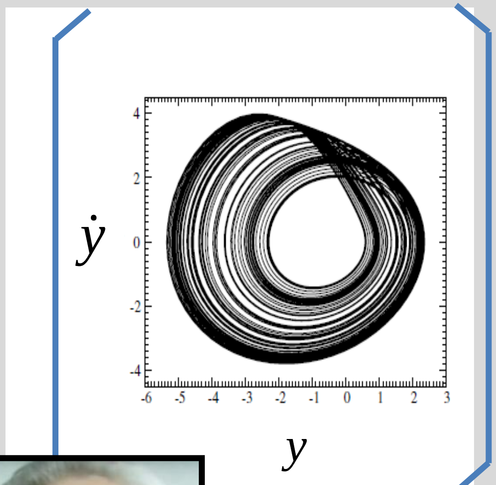


Mangiarotti et al. *Phys. Rev. E* (2012)

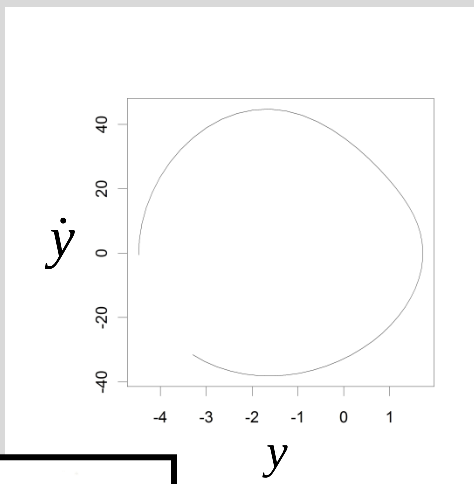
GPoM platform, the potential of the approach

Rössler-y

Original system



Unstable periodic orbits (e.g. period 1)



O. Rössler

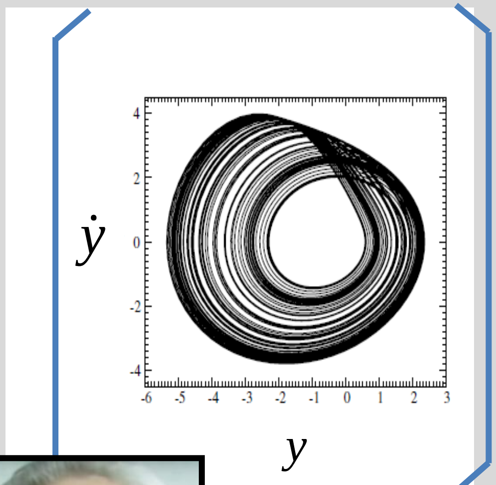
Letellier, Mangiarotti & Aguirre (under revision)

Letellier et al., Entropie 1997

GPoM platform, the potential of the approach

Rössler-y

Original system



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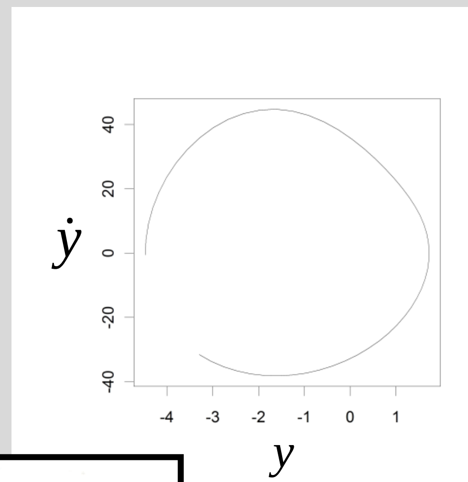
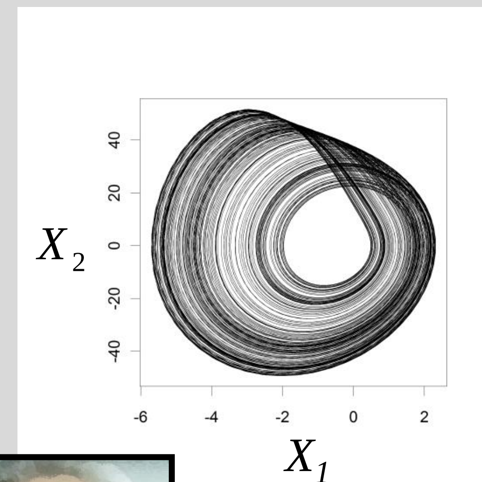


Global model



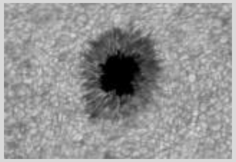
O. Rössler

Letellier, Mangiarotti & Aguirre (under revision)
Letellier et al., Entropie 1997

Global modeling

applied to real observations univariate

NARMA



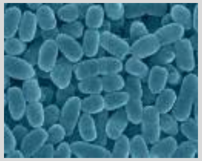
Aguirre et al. 2008

- sun
- epidemiology
- ecological
- agricultural
- climatic

.. systems

- sunspot cycles
- whooping caugh
- Canadian lynx
- cereal crops
- snow cycles

NARMA



Boudjema et Cazelles 2001

ODE

Maquet et al. 2007

ODE

Mangiarotti et al.
2011 RNL; 2014 *Chaos*

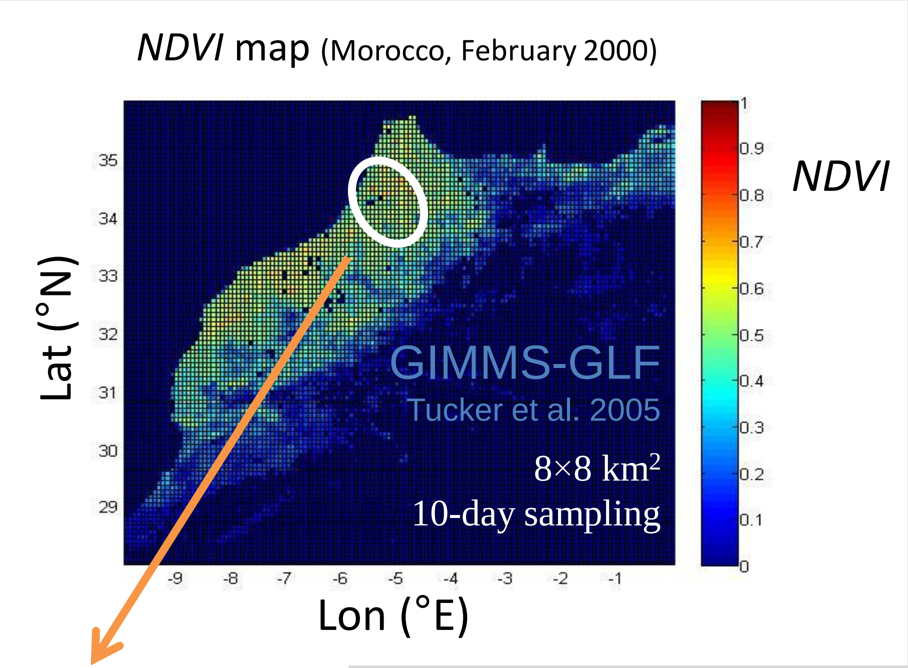
ODE

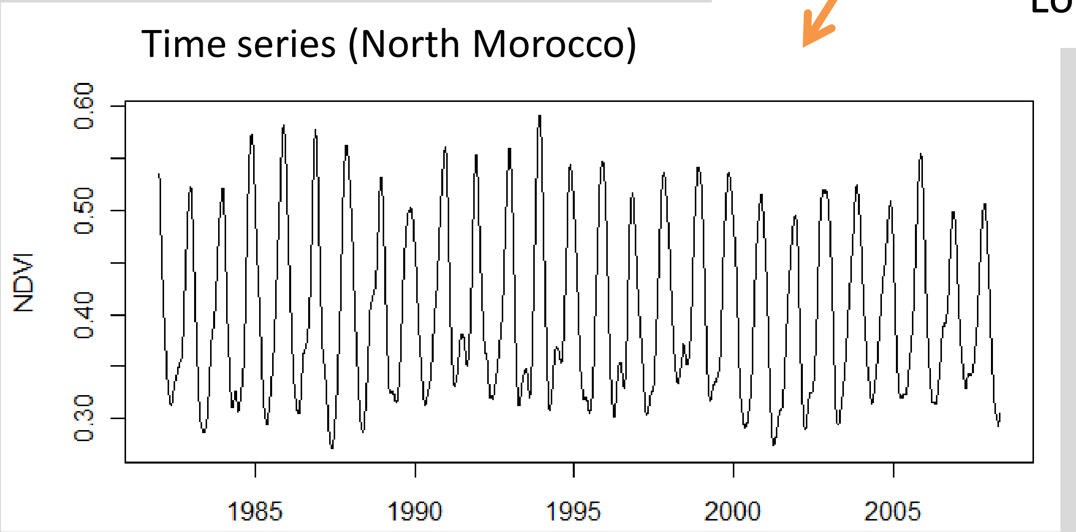
Mangiarotti 2014 *HDR*

Normalized Difference Vegetation Index

$$\mathrm{NDVI} = \frac{\rho_{PIR} - \rho_R}{\rho_{PIR} + \rho_R}$$

0.1 0.7



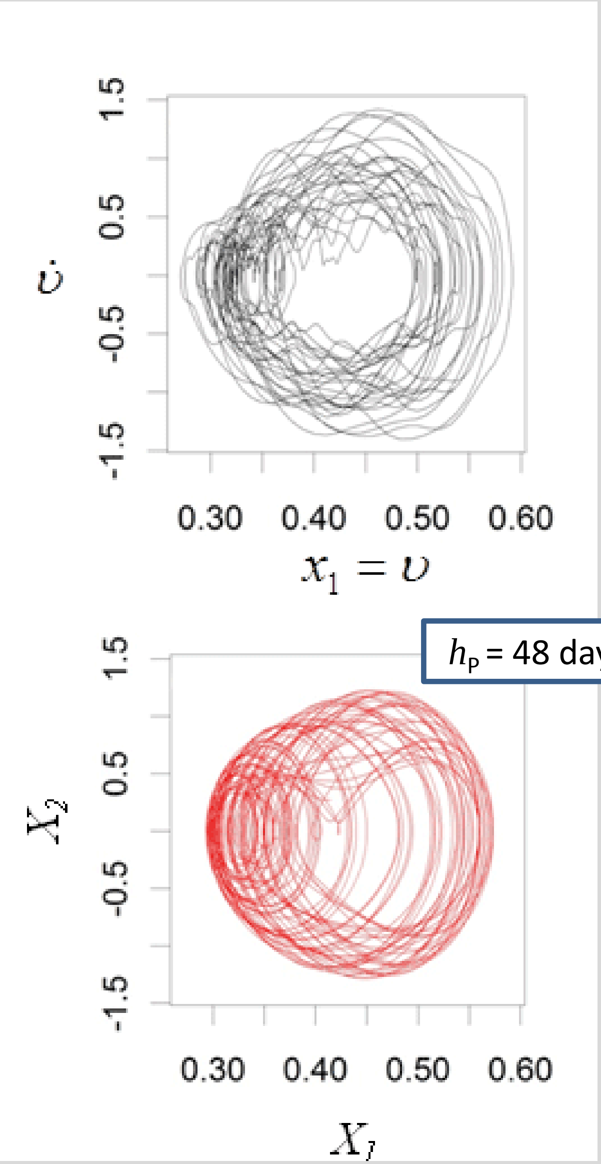


A global model for *cereal crops*

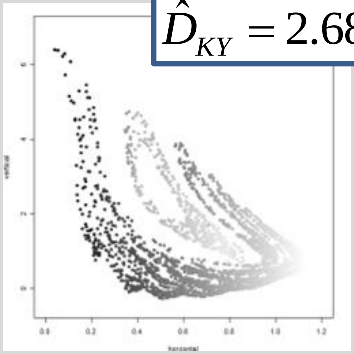
- Complex structure
- Very few previous cases
- Never directly obtained from data

data

model

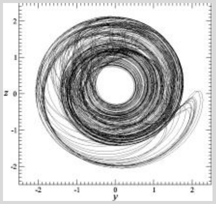


Poincaré section



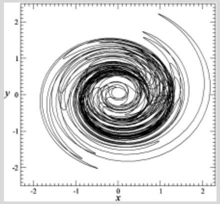
$$\hat{D}_{KY} = 2.68 >> 2$$

$\hat{D}_{KY} = 2.39$



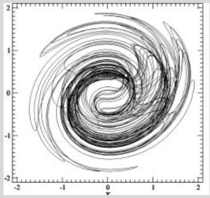
Lorenz-84

$\hat{D}_{KY} = 2.76$

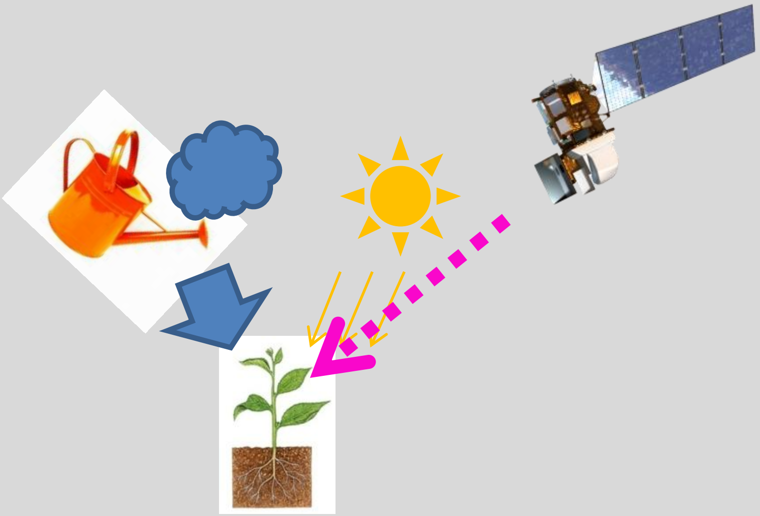


Wieczorek 1999

$\hat{D}_{KY} = 2.54$



Chlouverakis 2002



Chaos modelling applied to crops detection in South India

S. Mangiarotti, A.K. Sharma, Sekhar M., L.Ruiz, S. Corgne, L. Hubert-Moy, Y. Kerr

TOSCA

Crops'I Chaos
(2016-2017)

Irriga-detection
(2017-2019)

ATCHA
(2017-2019)

-Inde
-France (SO)
-Mexique

Chaos modelling applied to crops detection in South India

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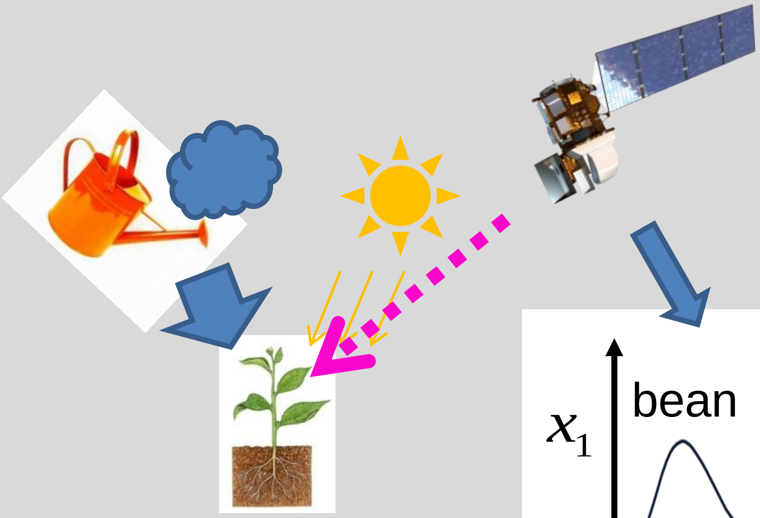
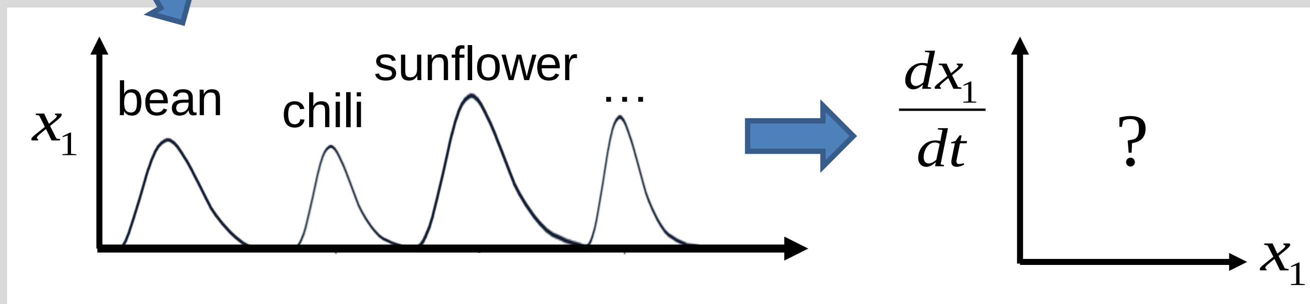
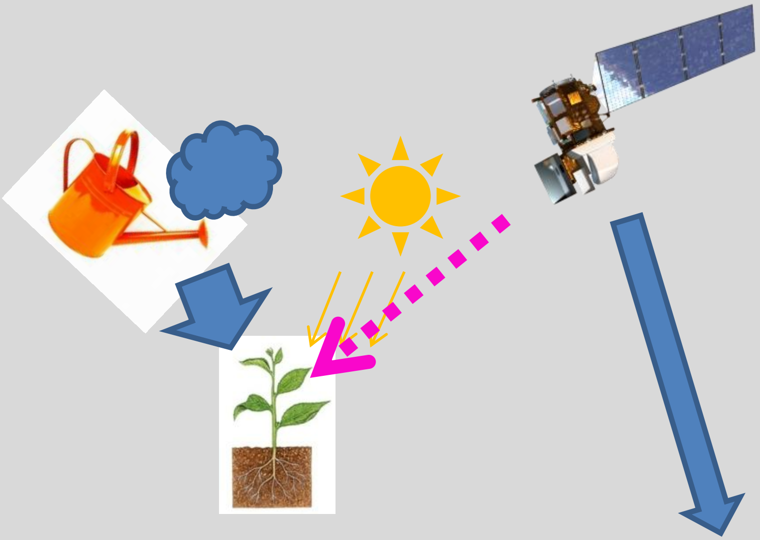




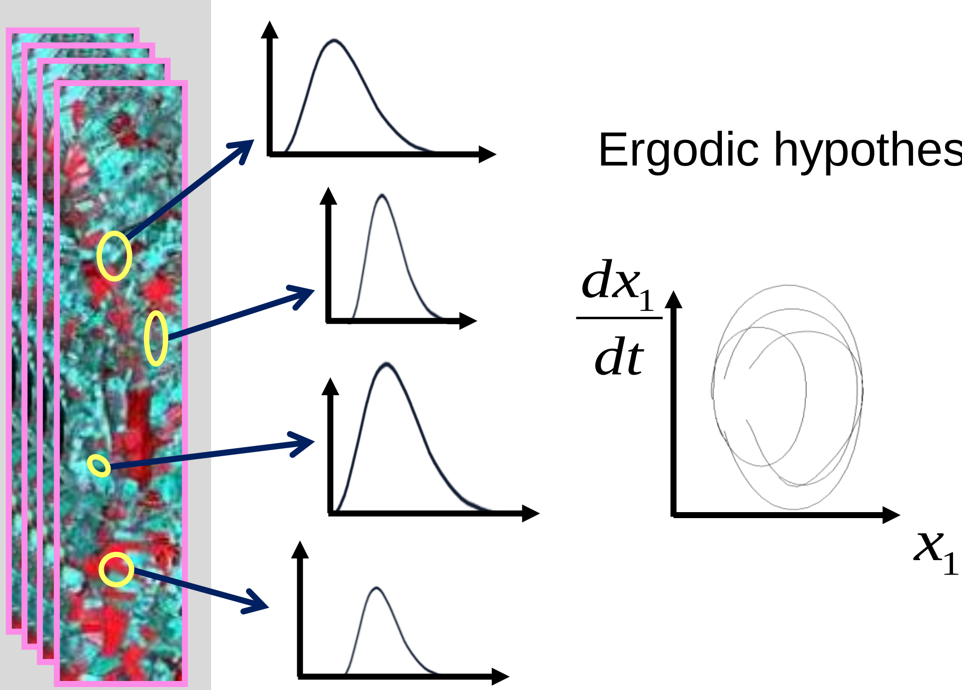
$$
\begin{cases}
\dot{x}_1 = f_1(x_1, x_2, \ldots, x_d) \\
\dot{x}_2 = f_2(x_1, x_2, \ldots, x_d) \\
\ldots \\
\dot{x}_d = f_d(x_1, x_2, \ldots, x_d)
\end{cases}
$$

Chaos modelling applied to crops detection in South India

S. Mangiarotti, A.K. Sharma, Sekhar M., L.Ruiz, S. Corgne, L. Hubert-Moy, Y. Kerr

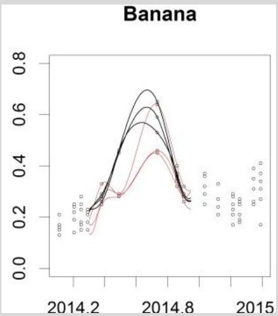


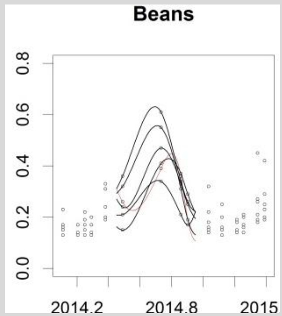
$$\begin{cases} \dot{x}_1 = f_1(x_1, x_2, \ldots, x_d) \\ \dot{x}_2 = f_2(x_1, x_2, \ldots, x_d) \\ \ldots \\ \dot{x}_d = f_d(x_1, x_2, \ldots, x_d) \end{cases}$$

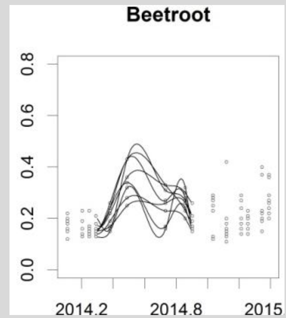


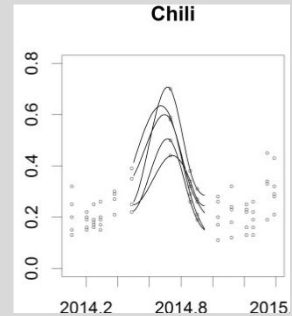
Global modelling

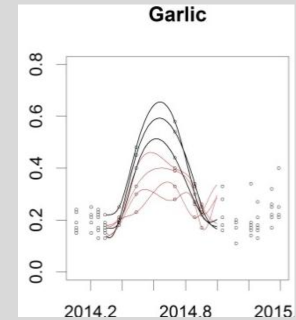
$$\begin{cases} \dot{x}_1 = X_2 \\ \dot{X}_2 = X_3 \\ \ldots \\ \dot{X}_n = R(x_1, X_2, \ldots, X_n) \end{cases}$$

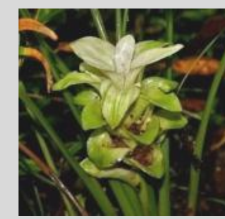

Banana
0.0 0.2 0.4 0.6 0.8
2014.2 2014.8 2015

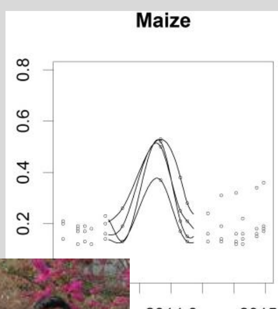

Beans
0.0 0.2 0.4 0.6 0.8
2014.2 2014.8 2015

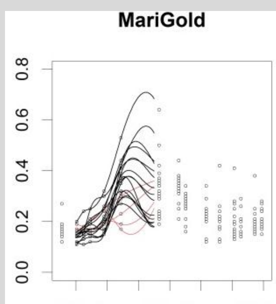

Beetroot
0.0 0.2 0.4 0.6 0.8
2014.2 2014.8 2015

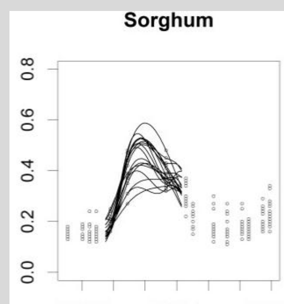

Chili
0.0 0.2 0.4 0.6 0.8
2014.2 2014.8 2015

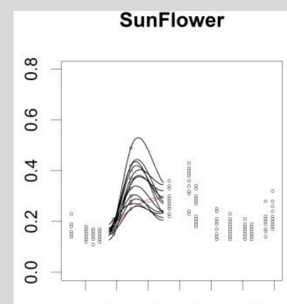

Garlic
0.0 0.2 0.4 0.6 0.8
2014.2 2014.8 2015

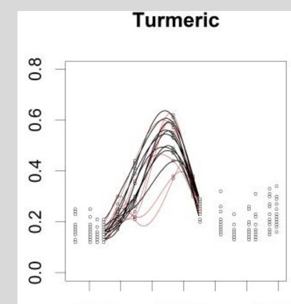


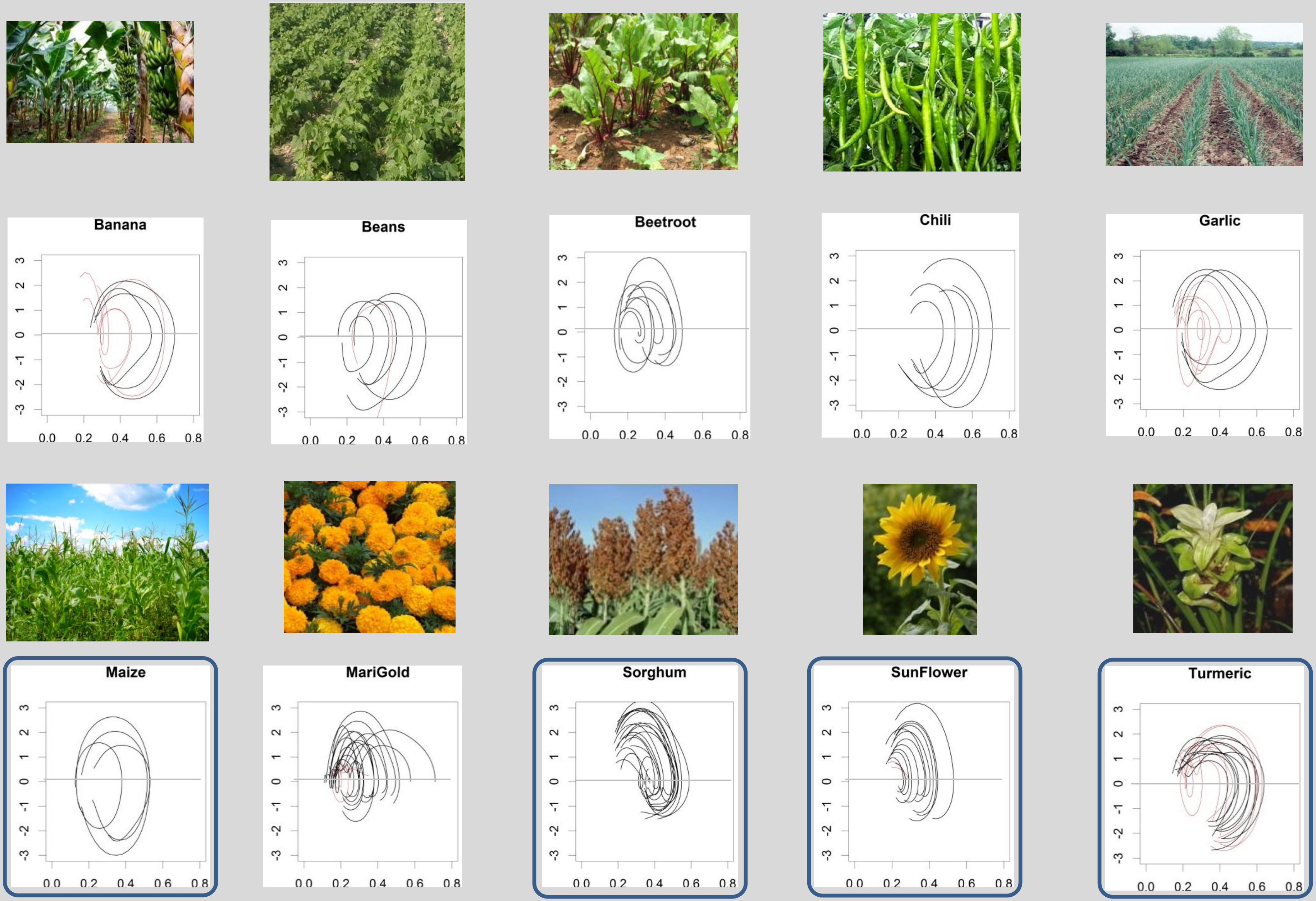

Maize
0.2 0.4 0.6 0.8
2014.8 2015


MariGold
0.0 0.2 0.4 0.6 0.8
2014.2 2014.8 2015


Sorghum
0.0 0.2 0.4 0.6 0.8
2014.2 2014.8 2015


SunFlower
0.0 0.2 0.4 0.6 0.8
2014.2 2014.8 2015


Turmeric
0.0 0.2 0.4 0.6 0.8
2014.2 2014.8 2015


Banana
Beans
Beetroot
Chili
Garlic
Maize
MariGold
Sorghum
SunFlower
Turmeric

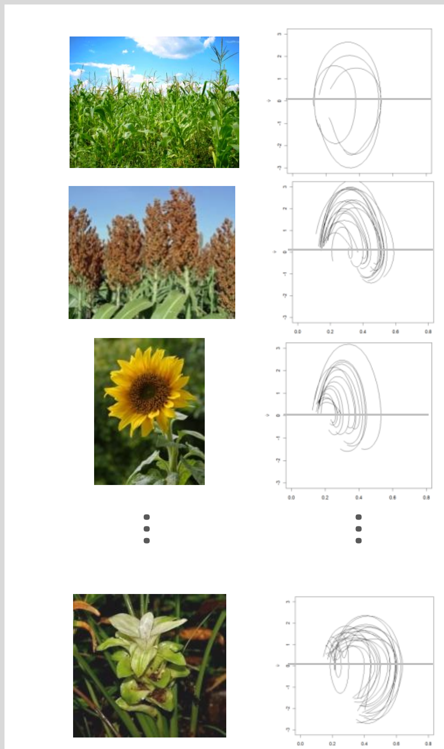
Maize

Sorghum

Sunflower

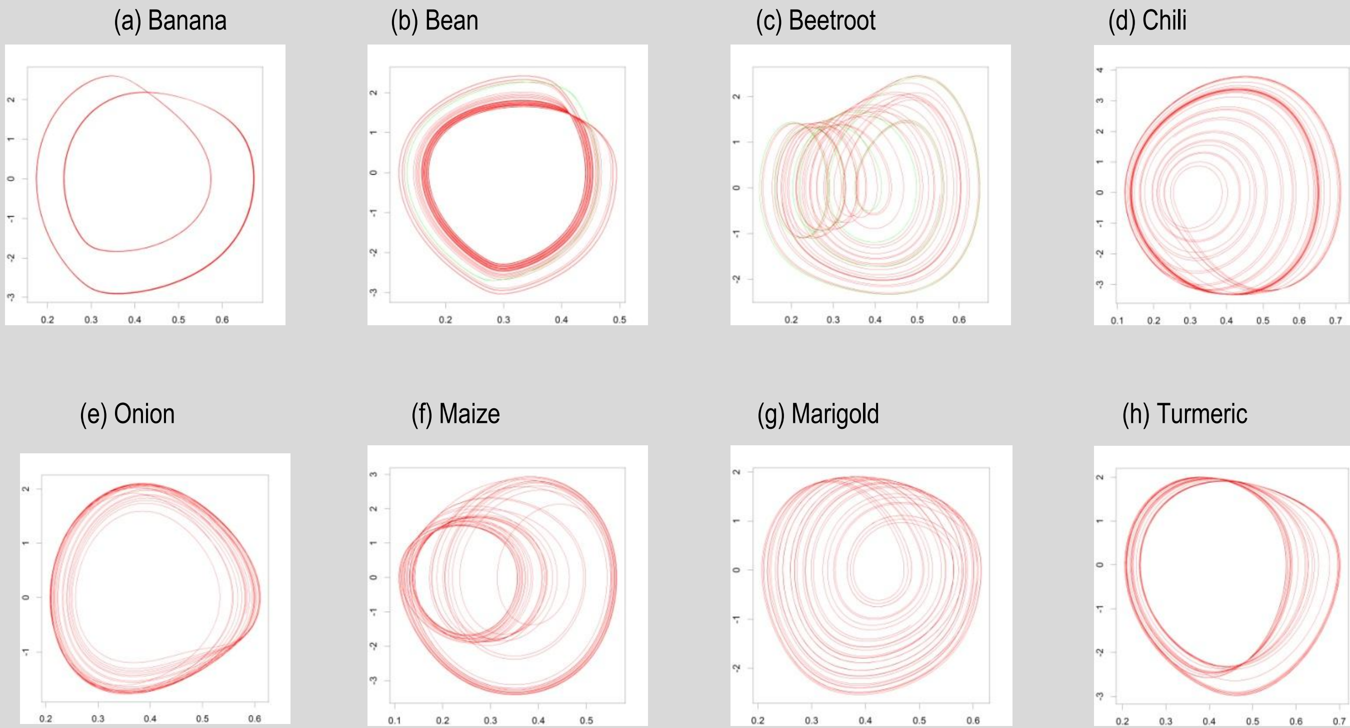
⋮

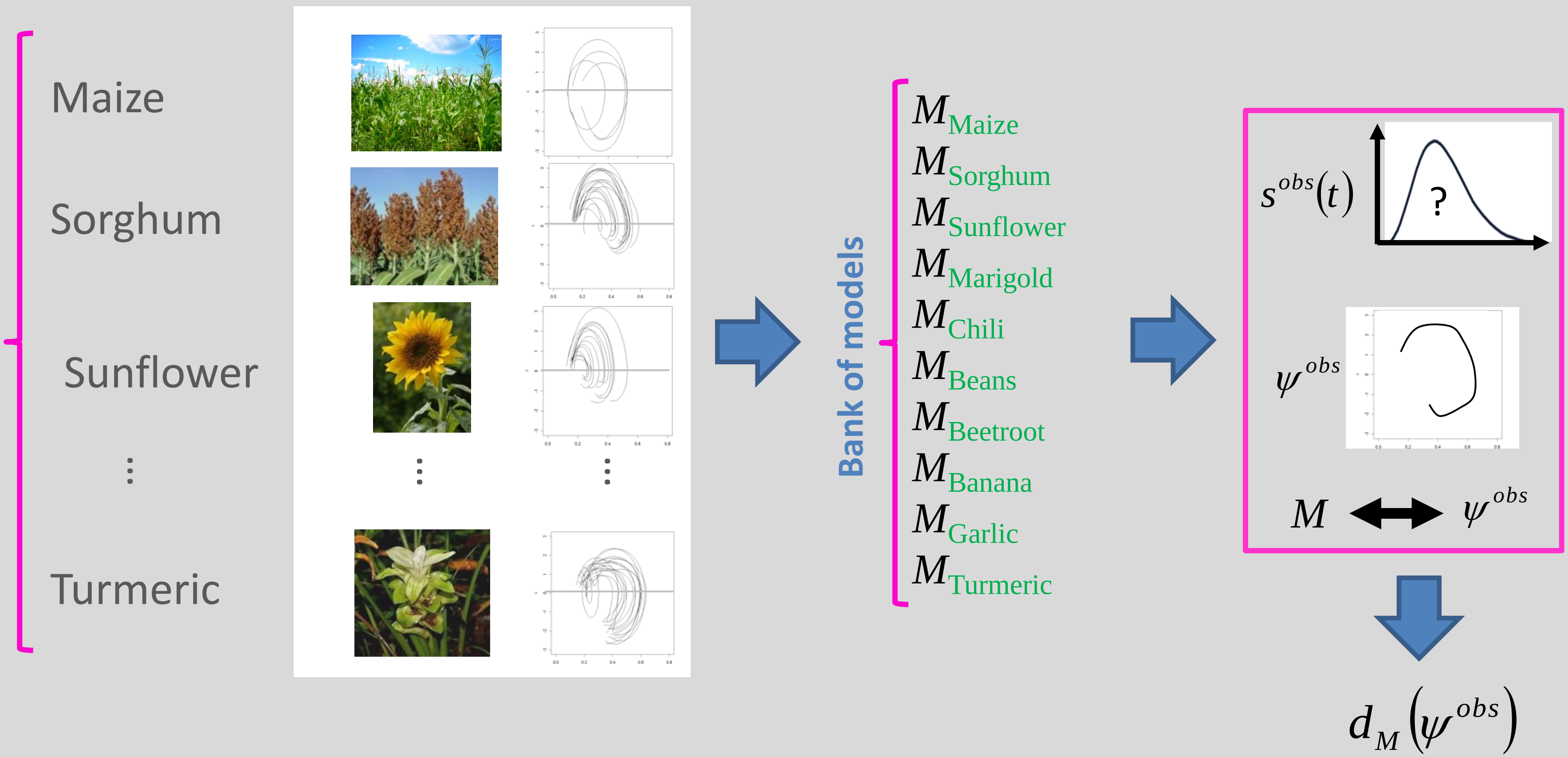
Turmeric



Bank of models

M_{Maize}
M_{Sorghum}
$M_{\text{Sunflower}}$
M_{Marigold}
M_{Chili}
M_{Beans}
M_{Beetroot}
M_{Banana}
M_{Garlic}
M_{Turmeric}


(a) Banana
(b) Bean
(c) Beetroot
(d) Chili
(e) Onion
(f) Maize
(g) Marigold
(h) Turmeric


Maize
Sorghum
Sunflower
Turmeric
Bank of models
M_{Maize}
M_{Sorghum}
$M_{\text{Sunflower}}$
M_{Marigold}
M_{Chili}
M_{Beans}
M_{Beetroot}
M_{Banana}
M_{Garlic}
M_{Turmeric}
$s^{obs}(t)$
?
ψ^{obs}
$M \leftrightarrow \psi^{obs}$
$d_M(\psi^{obs})$

Définition de la métrique

$$\begin{cases} \dot{x}_1 = X_2 \\ \dot{X}_2 = X_3 \\ \quad \ldots \\ \dot{X}_n = R(x_1, X_2, .., X_n) \end{cases}$$

Définition de la métrique

$$\begin{cases} \dot{x}_1 = X_2 \\ \dot{X}_2 = X_3 \\ \quad \cdots \\ \dot{X}_n = P(x_1, X_2, .., X_n) + \varepsilon \end{cases}$$

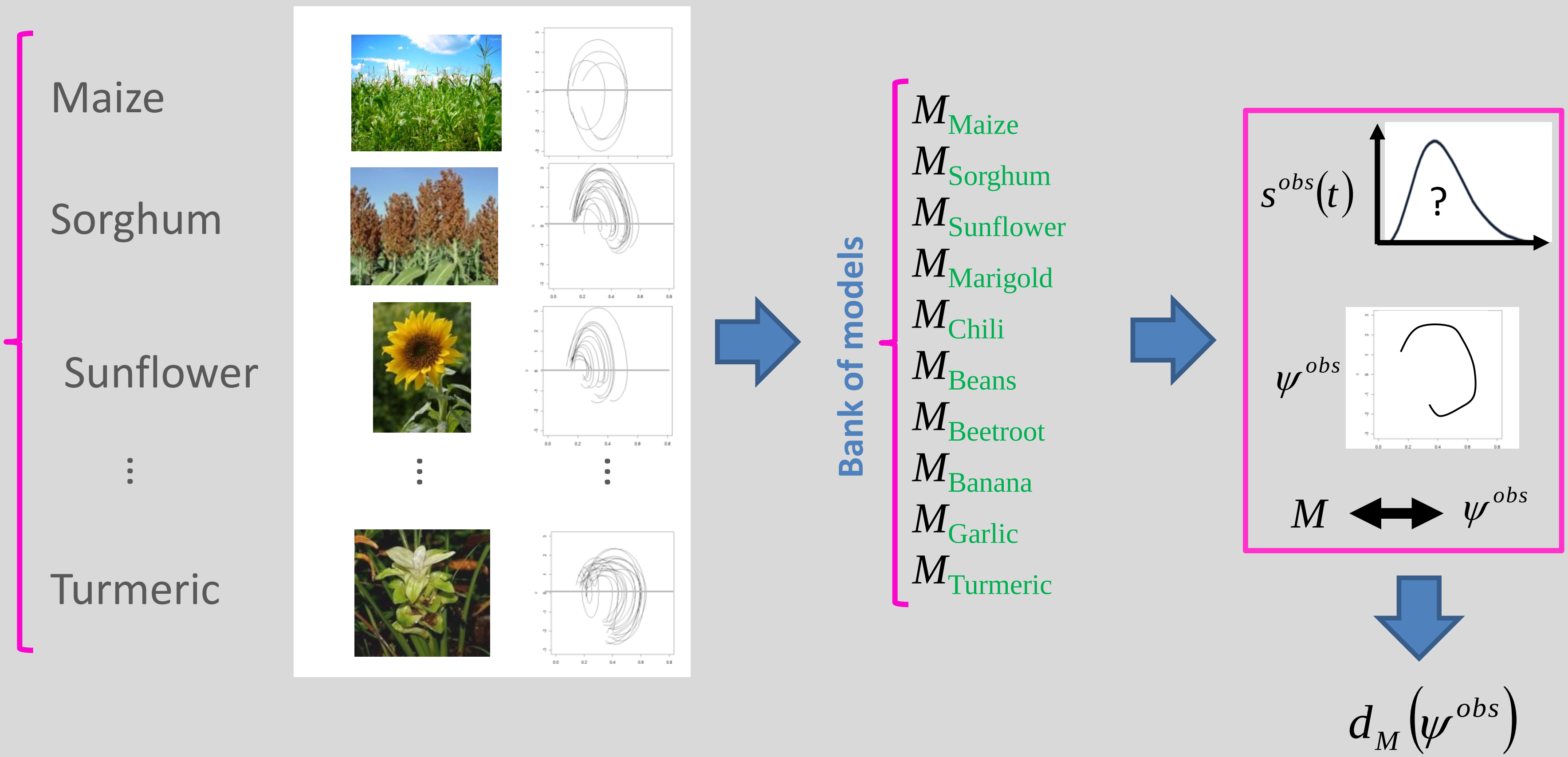
Quantité à minimiser

$$d_{M-\psi} = \left\| \dot{X}_n - P(x_1, X_2, .., X_n) \right\|_{[t;t+T]}$$

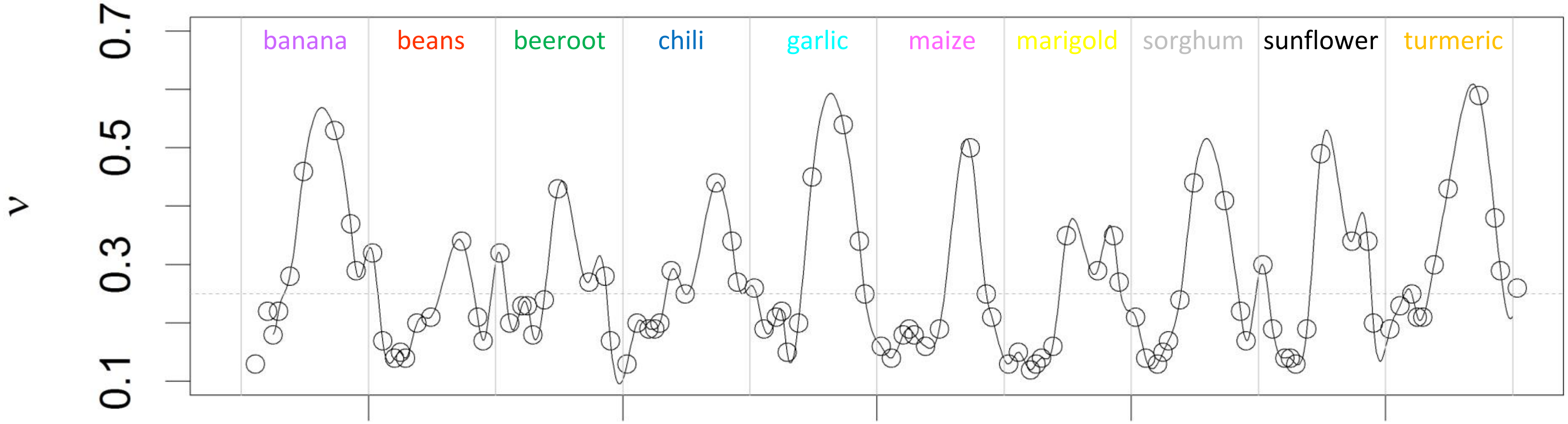
Définition de la métrique

- signal NDVI observé $s(t)$: $$\psi(s) = \left(s_1^{obs},\, s_2^{obs},\, s_3^{obs},\, ..,\, s_n^{obs}\right)$$
- un modèle M d'une culture donnée: $$P\left(X_1,\, X_2,\, X_3,\, ..,\, X_n\right)$$

$$\boxed{d_{M-\psi}(t) = \left\| \dot{s}_n^{obs} - P\left(s_1^{obs},\, s_2^{obs},\, ..,\, s_n^{obs}\right) \right\|_{[t;t+T]}}$$

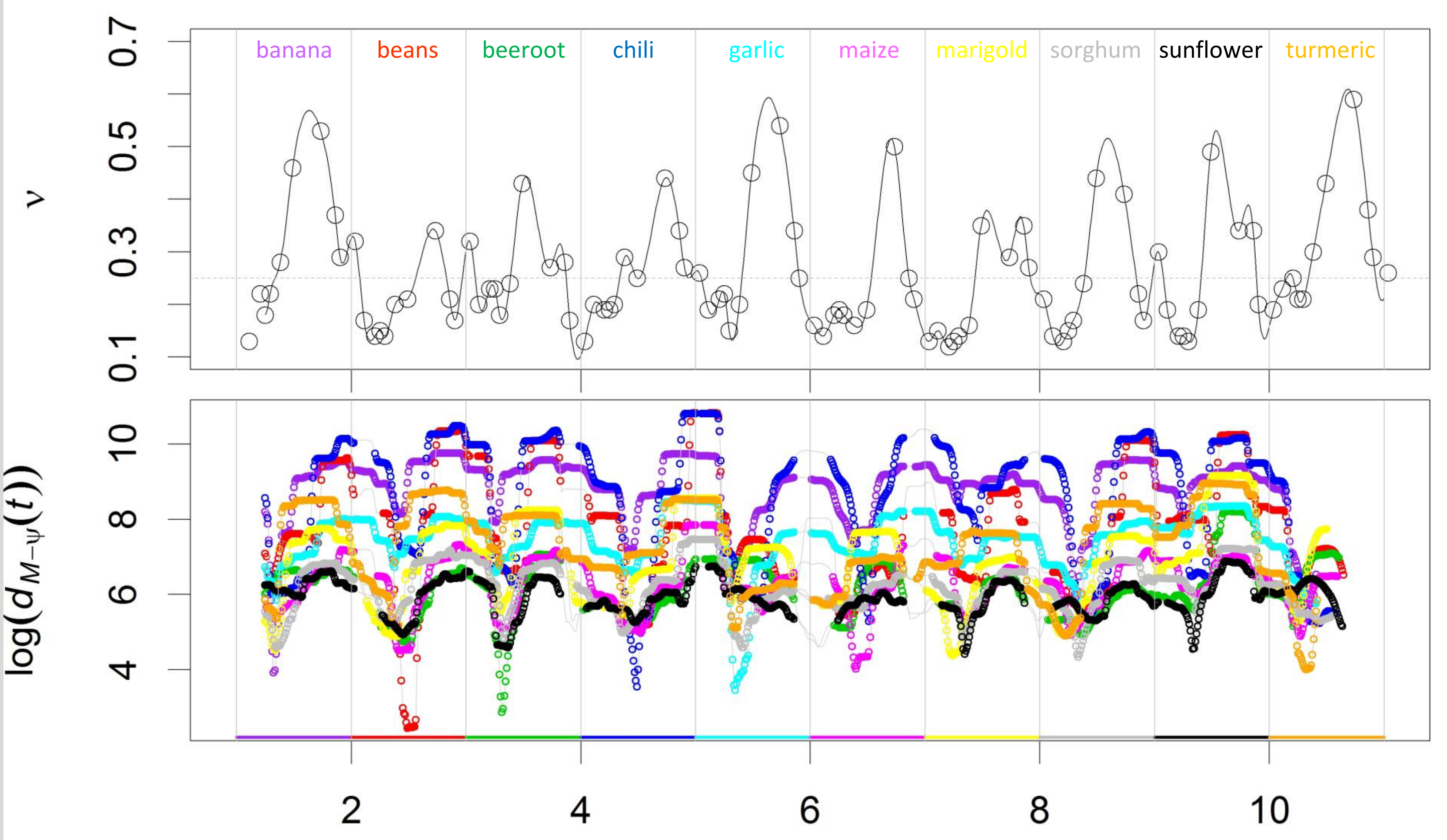


Maize
Sorghum
Sunflower
Turmeric
Bank of models
M_{Maize}
$M_{Sorghum}$
$M_{Sunflower}$
$M_{Marigold}$
M_{Chili}
M_{Beans}
$M_{Beetroot}$
M_{Banana}
M_{Garlic}
$M_{Turmeric}$
$s^{obs}(t)$
?
ψ^{obs}
$M \leftrightarrow \psi^{obs}$
$d_M(\psi^{obs})$

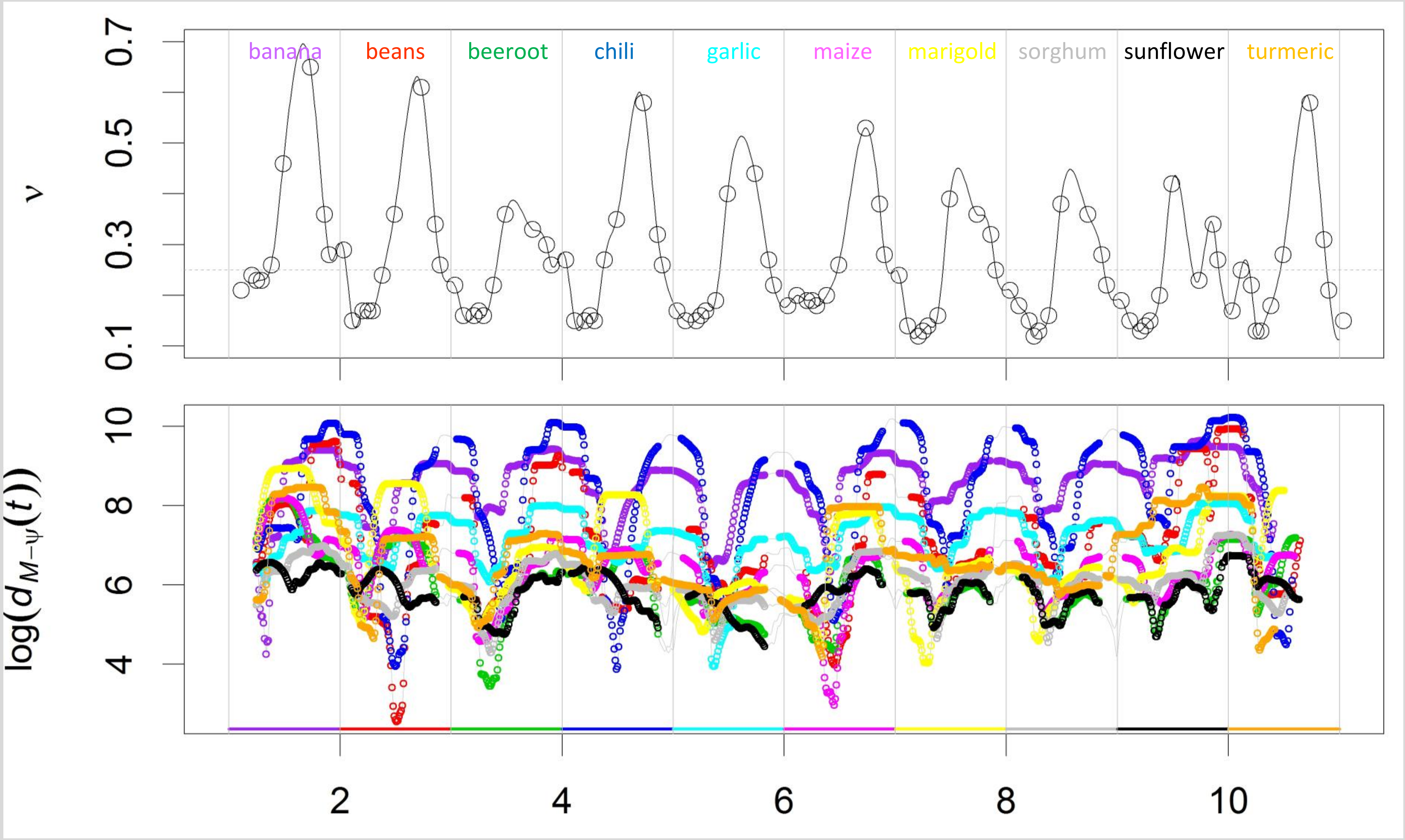


v
0.1
0.3
0.5
0.7
banana
beans
beeroot
chili
garlic
maize
marigold
sorghum
sunflower
turmeric

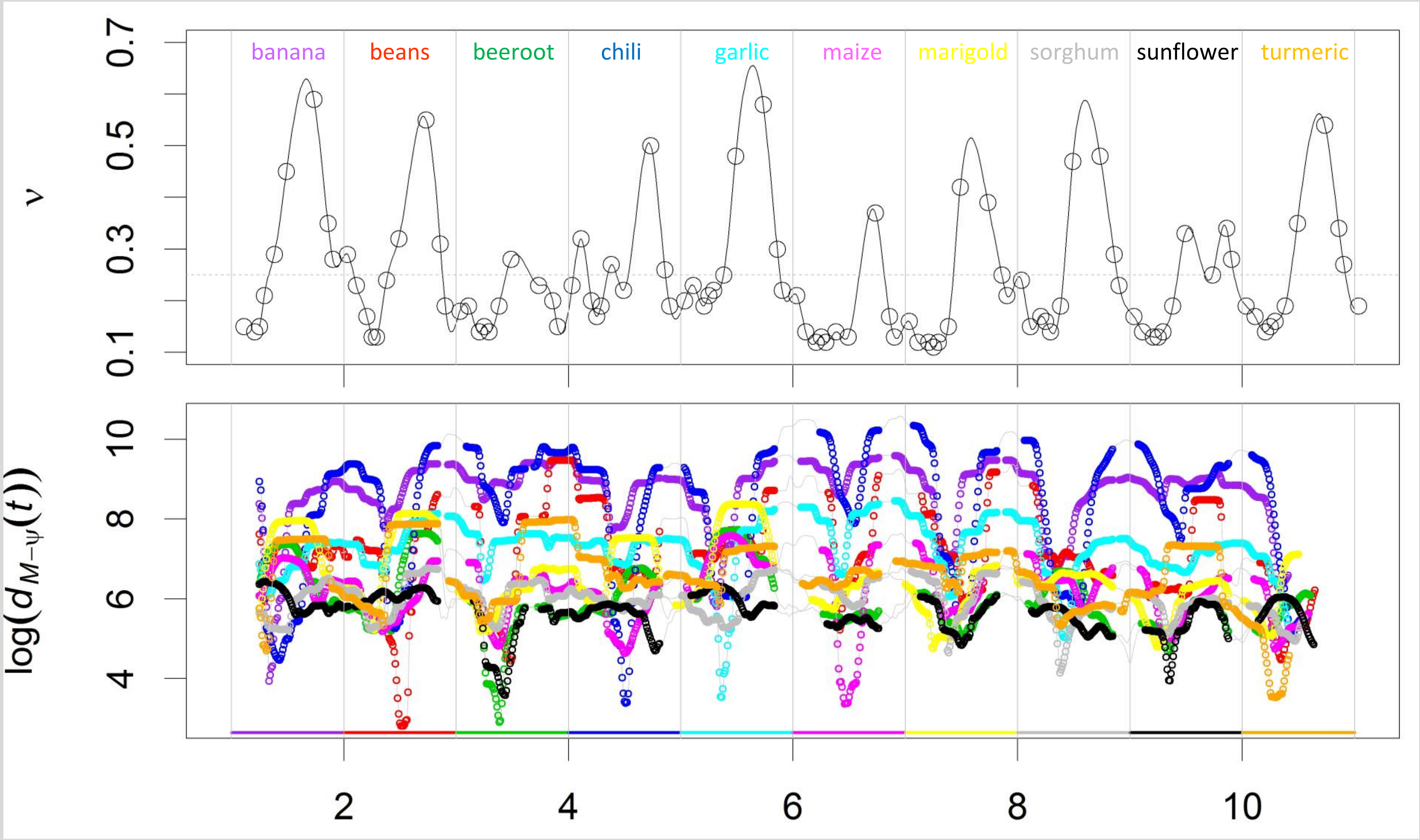
Crop detection (1)



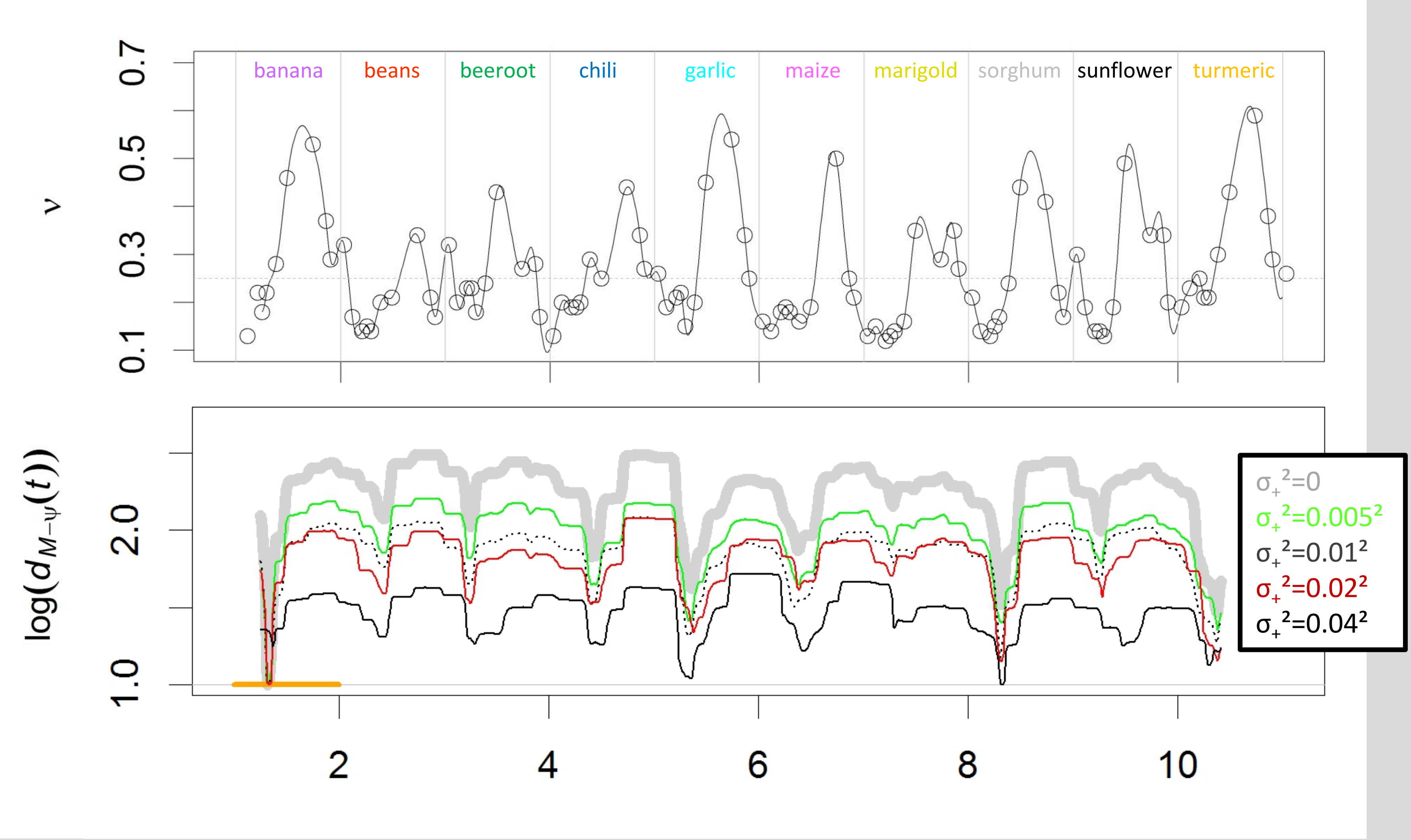
Crop detection (2)



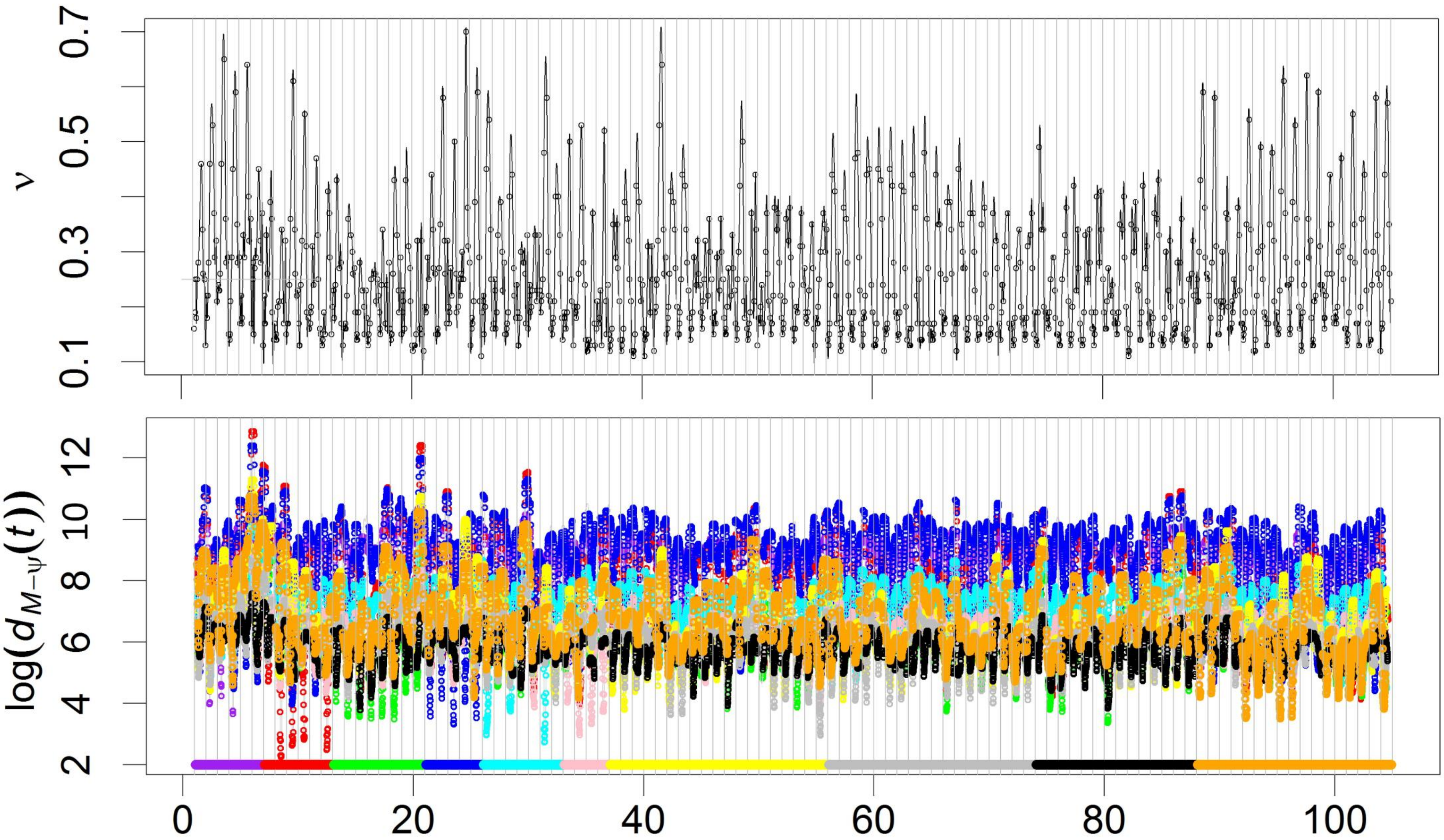
Crop detection (3)



Sensitivity to noise (banana-Crop)



Application to the whole data set (104 time series)



banana bean beeroot chili garlic maize marigold sorghum sunflower turmeric

Phase space

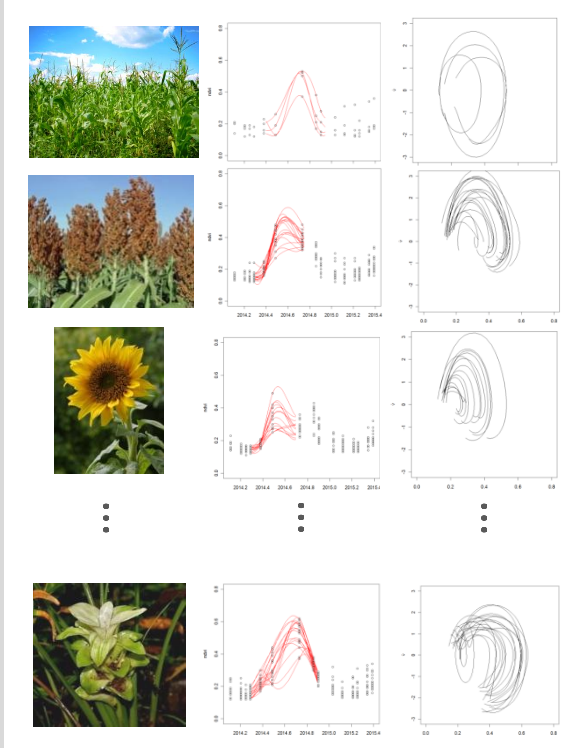
Maize

Sorghum

Sunflower

⋮

Turmeric



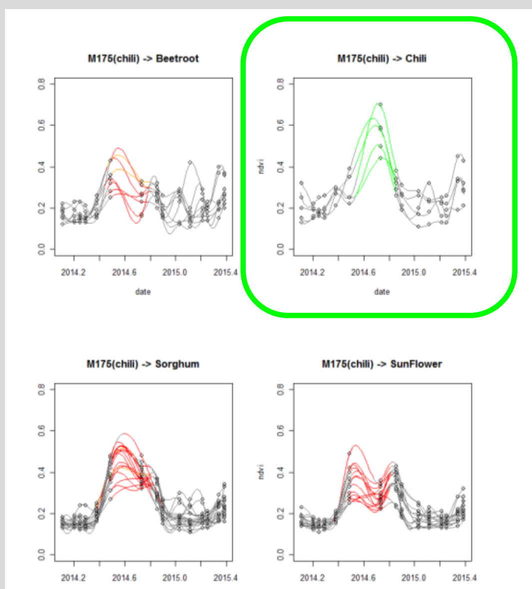
Bank of models

M_{Maize}
$M_{Sorghum}$
$M_{Sunflower}$
$M_{Marigold}$
M_{Chili}
M_{Beans}
$M_{Beetroot}$
M_{Banana}
M_{Garlic}
$M_{Turmeric}$

$$d\left(M, \psi^{obs}\right)$$

Example:

M_{Chili}



Detection

statistics

100% ok
(5/5)

4% erroneous
(4/104)

St. Johns Research Institute

2nd August 2016

Thank you for your interest